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**WECC CAISO SPECIFIC NEEDS FOR  
LOOP FLOW MONITORING,  
MANAGEMENT,  
NEAR TERM PREDICTION  
AND PROBABILISTIC ASSESSMENT,  
AND PROTOTYPE MONITORING SYSTEM  
DESIGN**

*Prepared For:*

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# **WECC CAISO Specific Needs for Loop Flow Monitoring, Management, Near Term Prediction and Probabilistic Assessment, And Prototype Monitoring System Design**

## **Technical Report**

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## Preface

The U.S. Electricity Grid Today The U.S. electric power system is in the midst of a fundamental transition from a centrally planned and utility-controlled structure to one that will depend on competitive market forces for investment, operations, and reliability management. Electricity system operators are being challenged to maintain the reliability of the grid and support economic transfers of power as the industry's structure changes and market rules evolve. Meanwhile, U.S. economy depends more than ever on reliable and high quality electricity supplies. New technologies are needed to prevent major outages such as those experienced on the Western grid on August 10, 1996, which left 12 million people without electricity for up to eight hours and cost an estimated \$2 billion.

The Consortium for Electric Reliability Technology Solutions (CERTS) was formed in 1999 to research, develop, and disseminate new methods, tools, and technologies to protect and enhance the reliability of the U.S. electric power system and functioning of a competitive electricity market. CERTS is currently conducting research for the U.S. Department of Energy (DOE) Transmission Reliability Program and for the California Energy Commission (CEC) Public Interest Energy Research (PIER) Program. The members of CERTS include the Electric Power Group, Lawrence Berkeley National Laboratory, Oak Ridge National Laboratory, Pacific Northwest National Laboratory, the National Science Foundation's Power Systems Engineering Research Center, and Sandia National Laboratories.

## TABLE OF CONTENTS

|   |    |
|---|----|
| ABSTRACT  | 5  |
| NOMENCLATURE  | 6  |
| CHAPTER   |    |
| 1 INTRODUCTION  | 12 |
| Motivation  |    |
| Objectives  |    |
| The accommodation / control of loop flows   |    |
| Transmission engineering and electricity markets                                      |    |
| Mathematics of estimation   |    |
| Game theory and application   |    |
| Sparse matrix methods for power flow algorithms                                       |    |
| MATLAB programming and visualization  |    |
| 2 LOOP FLOWS IN ELECTRIC POWER SYSTEMS  | 24 |
| Deregulation of electric power markets  |    |
| Electricity markets: types, structure, and operation                                  |    |
| What are loop flows?  |    |
| Types of loop flows   |    |
| Causes and effects of loop flows  |    |
| Contemporary practices relating to loop flows   |    |
| CHAPTER   |    |
| 3 MODEL BASED ESTIMATION OF LOOP FLOWS  | 37 |
| An introduction to state estimation models  |    |
| Model adequacy  |    |
| Conceptualization of the loop flow problem  |    |
| Methods of estimation   |    |
| Considerations in loop flow model verification  |    |
| 4 ACCOMMODATION OF UNSCHEDULED FLOWS IN A GENCO'S PERSPECTIVE                         | 48 |
| Need for accommodation of unscheduled flows   |    |
| A contribution factor for participating GENCOs  |    |
| Transmission pricing in deregulated electric power markets                            |    |
| Accommodation of USF using transmission pricing paradigms                             |    |
| USF accommodation algorithm   |    |
| Game theory   |    |
| Game theory based accommodation of USF among GENCOs                                   |    |
| 5 ILLUSTRATIVE CASE STUDIES OF LOOP FLOW ESTIMATION AND ACCOMMODATION IN TEST SYSTEMS | 58 |
| Introduction  |    |

|            |   |    |
|------------|---|----|
|            | Case study on a 9 bus test system                               |    |
|            | Case study on the modified IEEE 30 bus test system              |    |
|            | Case study on the IEEE 57 bus test system                       |    |
| CHAPTER    |   |    |
|            | 6 UNSCHEDULED FLOW ACCOMMODATION SOFTWARE PRO-<br>TOTYPE DESIGN | 77 |
| CHAPTER    |   |    |
|            | An expert system to accommodate unscheduled flows               |    |
|            | USFACC- A prototype GUI for accommodation USF                   |    |
|            | 7 CONCLUSIONS AND RECOMMENDATIONS                               | 81 |
|            | Conclusion  |    |
|            | Recommendations and future work                                 |    |
| REFERENCES |   |    |
| APPENDIX   |   |    |
|            | A DISCRETE KALMAN FILTERING                                     |    |
|            | B PRINCIPAL COMPONENTS ESTIMATION                               |    |
|            | C TRANSMISSION PRICING PARADIGMS                                |    |
|            | D CASE STUDY ON THE 9 BUS TEST SYSTEM                           |    |
|            | E CASE STUDY ON THE MODIFIED IEEE 30 BUS TEST SYSTEM            |    |
|            | F CASE STUDY ON THE IEEE 57 BUS TEST SYSTEM                     |    |

## ABSTRACT

This technical report addresses the original research of the investigators of the subcontract no. 6704263 toward prediction and management of loop flows in wide area competitive electric power systems. The research introduces a novel concept of accommodating the pre-congestion level loop flows in the system among participating generation companies (GENCOs) by designing a ‘take or pay’ charge for loading the transmission circuits with loop flows. This is a departure from the present industry practice of managing loop flows from transmission companies (TRANSCOs) perspective by a control and curtailment of schedules.

An innovative model describing the minor loop flows in the system in terms of the unscheduled flows (USF) on transmission circuits is developed and validated. A stringent statistical test based on the structure of the incidence matrix is devised for establishing a level of confidence in estimation. Several state estimation techniques uncommon to the field of electric markets are employed for estimation as well as validation purposes. This method shows considerable improvement and ease of calculation over the current industry methods of estimating loop flows. Also, this method is not empirical and heuristic as the present methods.

A novel formula for determining the individual contribution of every generating utility in a loop flow scenario is invented. The formula is shown to be used in conjunction with transmission pricing paradigms to obtain the respective penalty or compensation cost associated with each utility in the system.

An original algorithm is developed to tag schedules of utilities in a wide area system and determine the contribution of a utility based on state estimation methods using the novel contribution factor formula. The algorithm is designed to be transparent to the utilities and is intended to be used by a central agency or arbitrator like an ISO.

Finally, a user friendly menu driven graphical user interface (GUI) for assigning the take or pay charge for the loop flows among the GENCOs is designed as a prototype for loop flow estimation and management.

## NOMENCLATURE

|                  |  |
|------------------|--|
| $ \cdot $        | Magnitude of a vector quantity<br>Determinant of a matrix quantity                     |
| $\ \cdot\ $      | Euclidean norm of a vector quantity  |
| $\$i$            | price of transmission on branch $i$ of the network                                     |
| $a$              | Variable quantity representing state of system   |
| $A$              | magnitude of the spectral amplitude  |
| AC               | Alternating Current  |
| ACF              | Autocorrelation Function   |
| AEP              | American Electric Power Company  |
| ATC              | Available Transmission Capacity  |
| $b$              | Variable quantity representing state of system   |
| $B$              | Number of branches in a circuit<br>Rectangular matrix of time varying quantities       |
| BLUE             | Best Linear Unbiased Estimator   |
| $bwn$            | Bandlimited white noise  |
| $c$              | Variable quantity representing state of system   |
| C                | Core of a $n$ -person game   |
| CAISO            | California Independent System Operator   |
| CERTS            | Consortium for Electric Reliability Technology Solutions                               |
| $CF_m$           | Contribution factor of utility $m$ toward USF in system                                |
| $CF_{mn}$        | Contribution factor of utility $m$ toward minor loop flow $n$                          |
| $C_{m,n,t}$      | Cost for transmitting power between adjacent nodes $m$ and $n$ for the transaction $t$ |
| $col_i(A_{jxi})$ | Column $i$ of matrix $A$ of order $jxi$  |



|              |   |
|--------------|---|
| Cond ( $H$ ) | Condition number of matrix $H$  |
| cov ( $s$ )  | Covariance of quantity $s$  |
| $C_t$        | Price of transaction $t$  |
| $D$          | Canonical form of matrix $H$  |
| DISTCO       | Distribution company  |
| $D_{m,n}$    | Cost per/MW-mile for transmitting one unit of power through one mile between adjacent nodes $m$ and $n$ |
| $e$          | Error in estimation   |
| $e_k^-$      | Error in recursive estimation at time instant $t_k$   |
| $E[X(t)]$    | Expected value of random process $X(t)$   |
| $F$          | Rectangular matrix of time varying elements   |
| FACTS        | Flexible AC Transmission Systems  |
| FERC         | Federal Energy Regulatory Commission  |
| $F_{m,n}$    | Total power flowing between adjacent nodes $m$ and $n$  |
| $F_{m,n,t}$  | Amount of power in MW flowing between adjacent nodes $m$ and $n$ during the transaction $t$             |
| FPC          | Federal Power Commission  |
| $G$          | Rectangular matrix of time varying elements   |
| $G_{22}$     | Self-conductance of the receiving bus 2   |
| GAPP         | General Agreement on Parallel Paths   |
| Gen( $m$ )   | Generation of utility $m$   |
| GENCO        | Generation company  |
| GM           | Gauss-Markov  |
| GUI          | Graphical User Interface  |
| $H$          | Process or incidence matrix   |

|             |   |
|-------------|---|
| $H^{-1}$    | Inverse of matrix $H$   |
| $H^T$       | Transpose of matrix $H$   |
| $I$         | Identity matrix   |
| ICA         | Independent Contract Administrator                                      |
| IEEE        | Institute of Electrical and Electronics Engineers                       |
| IPF         | Interface Participation Factor  |
| IPP         | Independent Power Producer  |
| ISO         | Independent System Operator   |
| ITCF        | Interutility Transmission Coordination Forum                            |
| $k$         | Biasing parameter in ridge estimation                                   |
| KF          | Kalman filter   |
| $K_k$       | Kalman gain at time $t(t)$  |
| $L$         | Number of meshes in a planar system                                     |
| $L_{m,n}$   | Length in miles of transmission line between adjacent nodes $m$ and $n$ |
| $L_p$       | $p^{th}$ Hölder norm  |
| LRIC        | Long run incremental cost method  |
| LRMC        | Long run marginal cost method   |
| LSM         | Least Squares Method  |
| $m$         | Minor loop flow index   |
| $M$         | Major loop flow index   |
| MatLab      | Matrix Laboratory software  |
| MECS        | Michigan Energy Coordinating System                                     |
| M-estimator | Maximum likelihood estimator  |

|              |   |
|--------------|---|
| MISO         | Midwest ISO   |
| $n$          | Number of minor loop flows in a system  |
| $N$          | Number of nodes in a circuit  |
| NYISO        | New York ISO  |
| OASIS        | Open Access Same-time Information Systems   |
| $p(e)$       | Likelihood function of quantity $e$   |
| $P_2$        | Power delivered to the receiving bus 2  |
| PCE          | Principal Components Estimation   |
| PDF          | Probability Density Function  |
| PJM          | Pennsylvania- New Jersey- Maryland  |
| $P_k$        | Error covariance matrix at time instant $t_k$                                       |
| $P_k^-$      | <i>a priori</i> estimate of error covariance matrix at time instant $t_k$           |
| $P_{peak}$   | Entire system load during peak condition  |
| $P_t$        | Load served during transaction $t$  |
| PTDF         | Power Transfer Distribution Factor  |
| PURPA        | Public Utilities Regulatory Policies Act  |
| $PX_t$       | Mile-MW value of the transaction $t$  |
| $Q_k$        | Variance of white noise driving function at time instant $t_k$                      |
| $r$          | Residual in estimation  |
| Rank ( $H$ ) | Rank of matrix $H$  |
| $R_k$        | Variance of white noise sequence associated with measurements at time instant $t_k$ |
| RTO          | Regional Transmission Organization  |

|                   |  |
|-------------------|--|
| $R_X$             | Autocorrelation function of $X(t)$   |
| S                 | Coalition of players in a $n$ -person game   |
| SNR               | Signal to Noise Ratio  |
| SRIC              | Short run incremental cost method  |
| SRMC              | Short run marginal cost method   |
| STM               | State Transition Matrix  |
| $Sum(Gen)$        | Sum of all generation present in the system  |
| SVD               | Singular Value Decomposition   |
| $S_{wn}(j\omega)$ | spectral amplitude of the white noise in the frequency domain                                    |
| $t$               | Tuning constant for robust function  |
| $T$               | Modal matrix of $H$  |
| $TC$              | Transmission charges   |
| TLR               | Transmission-line Loading Relief   |
| TPF               | Transaction Participation Factor   |
| TRANSCO           | Transmission company   |
| UPFC              | Unified Power Flow Controller  |
| USF               | Unscheduled Flow   |
| $USF\$_m$         | monetary value associated with the $m^{\text{th}}$ utility for participating in the USF scenario |
| $ V_1 $           | Magnitude of voltage at the sending bus  |
| $ V_2 $           | Magnitude of voltage at the receiving bus  |
| VIF               | Variance Inflation Factor  |
| $v_k$             | White noise sequence assumed to contaminate the observables at time instant $t_k$                |
| $w$               | column vector of white noise   |

|                    |   |
|--------------------|---|
| $W$                | physical frequency bandwidth  |
| WECC               | Western Electricity Coordinating Council  |
| WSCC               | Western Systems Coordinating Council  |
| $x$                | Vector of true states of a system<br>Vector of minor loop flows in a system                 |
| $\hat{x}$          | Vector of estimated states of a system obtained using LSM                                   |
| $\hat{x}_k^-$      | Best <i>a priori</i> estimate of system state at time instant $t_k$                         |
| $\dot{x}$          | Time differential of states in state-space model<br>Time derivative of process variable $x$ |
| $\hat{x}_{ridge}$  | Vector of estimated states of a system obtained using ridge estimation                      |
| $\hat{x}_{robust}$ | Vector of estimated states of a system obtained using robust regression                     |
| $\hat{x}_{PCE}$    | Vector of estimated states of a system obtained using Principal Components Estimation       |
| $X$                | Line reactance ( $\Omega$ )   |
| $X(t)$             | Process $X$ at time instant $t$   |
| $X/R$              | Reactance to resistance ratio of transmission circuit                                       |
| $x_{all\ n}$       | $n^{\text{th}}$ estimated minor loop flow in the aggregate schedule                         |
| $x_{i\ n}$         | $n^{\text{th}}$ estimated minor loop flow in the $i^{\text{th}}$ transaction set            |
| $y$                | Linear combination of system state variables  |
| $z$                | Vector of measurements (observations)   |
| $z_{agg}$          | $(B \times 1)$ vector of USF in the system during the aggregate schedule                    |
| $z_k$              | column vector of observables or measurement   |
| $v(n)$             | Characteristic function of a n-person game  |

|                           |   |
|---------------------------|---|
| $v(S)$                    | Total worth of the coalition $S$  |
| $v(S)-v(S\setminus\{i\})$ | Extra amount player $i$ brings to a coalition $S$                             |
| $v(S\setminus\{i\})$      | Worth of the coalition in the absence of player $i$ .                         |
| $\alpha$                  | Canonical form of system states $x$   |
| $\hat{\alpha}$            | Estimated values of canonical states  |
| $\hat{\alpha}_{PCE}$      | Reduced set of principal component estimates                                  |
| $\gamma$                  | Argument associated with the mutual admittance                                |
| $\tau$                    | Time difference variable of the GM process, $X(t)$ .                          |
| $\psi(e)$                 | Influence function of quantity $e$  |
| $\beta^{-1}$              | Time constant   |
| $\delta$                  | Difference between voltage angles at the terminals (r)                        |
| $\Lambda$                 | Diagonal matrix of the eigenvalues $[\lambda_1, \lambda_2, \dots, \lambda_p]$ |
| $\Delta(s)$               | Change in quantity ( $s$ )  |
| $\lambda$                 | Eigenvalue  |
| $\sigma^2(s)$             | Variance of quantity $s$<br>Mean square value of process $s$                  |
| $\mathfrak{F}(f_x(t))$    | Fourier transform of function $f(t)$  |
| $\phi_i(v)$               | Marginal contribution or the Shapley value of a player $i$                    |
| $\phi_k$                  | STM at time instant $t_k$   |

## CHAPTER 1 INTRODUCTION

### 1.1 Motivation

The primary motivation for this research is the requirement for some compensatory method to deal with the problem of Unscheduled Flows (USF) in wide area electric power systems like the Western Electricity Coordinating Council (WECC) (formerly known as Western Systems Coordinating Council (WSCC)). Power is desired to flow point-to-point in the wide area system according to transmission schedules prearranged by a central agency such as an Independent Contract Administrator (ICA) or an Independent System Operator (ISO). However, physical laws such as the Ohm's law and the Kirchhoff's laws may force the power flow to deviate from the scheduled paths. The path digressions are also compounded by changes in network topologies and other transaction schedules. The phenomenon of power flow deviating from the prearranged schedules is termed *unscheduled flow*. The USF are assumed to be caused by a phenomenon termed *loop flows*. It is also sometimes referred to as *parallel flow* and *circulating flow* [1]. Loop flows may pose potential hazards to the system operation like reduction in Available Transmission Capacity (ATC), limitation of transaction schedules, flow path congestion, forced participation in power transfer, deviation from prices leading to market pricing complications, overloading of lines causing security and reliability issues, and uncompensated loss for third parties. The control of loop flows can be achieved by employing dedicated devices such as Flexible AC Transmission Systems (FACTS), flow gates, and phase shifting transformers which may be cost prohibitive. Also, not all transmission circuits can be equipped with such dedicated devices. A selective procedure of installing these devices might introduce monopoly issues and some market pricing difficulty. Some of the control methods may add nonlinearity to the system. An alternative to the control of loop flows is the design of pricing strategies to accommodate them.

A strong motivation for the research work was the interest expressed by the Consortium for Electric Reliability Technology Solutions (CERTS) and the California Independent System Operator (CAISO) to develop better methods to predict loop flows on the systems based on historical data and probabilistic assessment. The CERTS management steering committee has also expressed interest in alternative methods for management of loop flows established on market based power sales [3]. Application of enhanced state estimation and regression techniques to the deregulated electricity markets, a new avenue of research initiative, also served as further motivation.

Pricing loop flows is a complicated process and may involve changes to the prices already set in a day-ahead or hour-ahead market. The industry currently practices methods that are empirical and potentially numerically cumbersome and have a disadvantage in addressing the issue on selected circuits where the loop flows have historically been observed to occur. Also, this method is designed from the perspective of the transmission companies (TRANSCOs) and does not penalize or compensate participating generation companies (GENCOs). Hence, a method which calculates the effects of loop flows on all the transmission circuits in the systems and estimates a contribution factor for accommodating the loop flows among the participating utilities (GENCOs) is left desired. The research work documented in this report describes the development of an equitable market pricing methodology for accommodating unscheduled flows in wide area systems from a GENCO perspective. The results of this work are presented as a menu-driven Graphical User Interface (GUI) which assists in displaying the loop flow data to the operators.

## 1.2 Objectives

The primary objective of this research work is to establish a method for estimating the loop flows that occur in a wide area system and to utilize the estimates to accommodate loop flows. The accommodation of loop flows is to be done utilizing the ‘power’ of the energy market, rewarding the participants that limit loop flow and penalizing the players that create loop flow. For this purpose, a linear mathematical model and a state space model based on the Gauss-Markov process representing the loop flows in the system is developed. Several state estimation and regression techniques are utilized to estimate the loop flows. As is required of any estimation problem, rigorous model validation procedures are performed. The structure of the data poses restrictions on the traditional methods of model validation. Hence, novel verification techniques based on traditional methods and advanced regression procedures are employed. The estimated loop flows are then intended to be used in designing an equitable market methodology for pricing the GENCOs participating in the loop flow scenario. In this way, the electricity market itself is used to accommodate the loop flows. The research aims at conceiving an expert system based on enhanced state estimation techniques for predicting the loop flows in the system. The expert system also incorporates a built-in power flow program which is based on sparse matrix methodologies. This method is intended to be user friendly and computationally efficient in order to make decisions regarding the loop flows such as controlling them or pricing the utilities responsible for causing parallel flows in the system. The secondary objectives of this research work are:

- To implement a power flow program based on sparse matrix technologies
- To interface the estimator machine with the power flow algorithm
- To develop a pricing methodology for accommodation of the loop flows
- To issue signals to utilities for controlling the USF
- To develop a user friendly interface for the entire process
- To provide a comprehensive documentation on loop flows.

Fig. 1.1 represents a Gantt chart of the tasks associated with the research on loop flows [3].

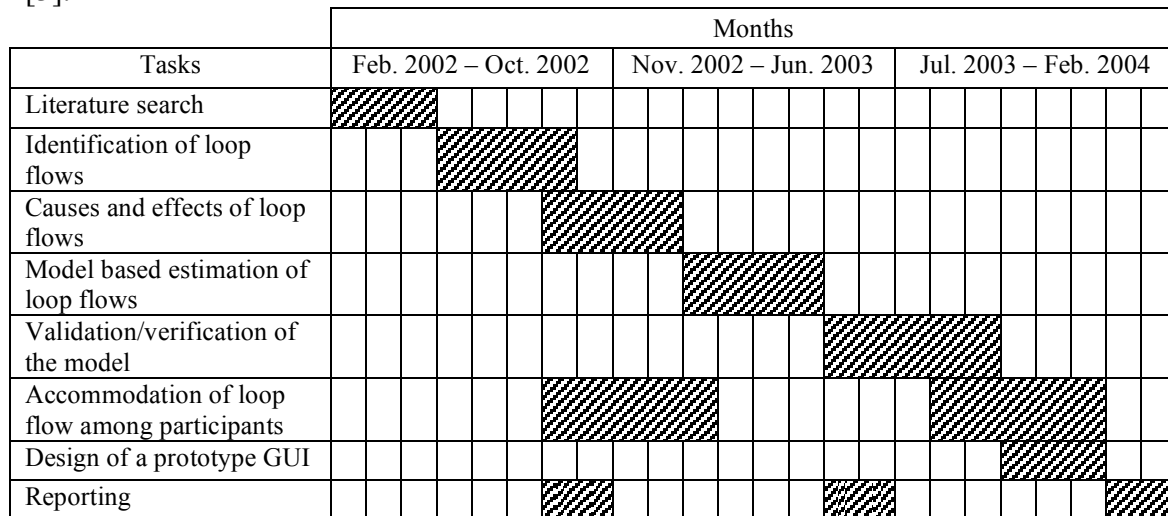


Fig. 1.1 Gantt chart of the task schedule in the loop flow research



A flowchart of the various task modules in the process of estimation and accommodation of loop flows in a wide area system is depicted in Fig. 1.2. The individual task sub-modules incorporate the secondary objectives.

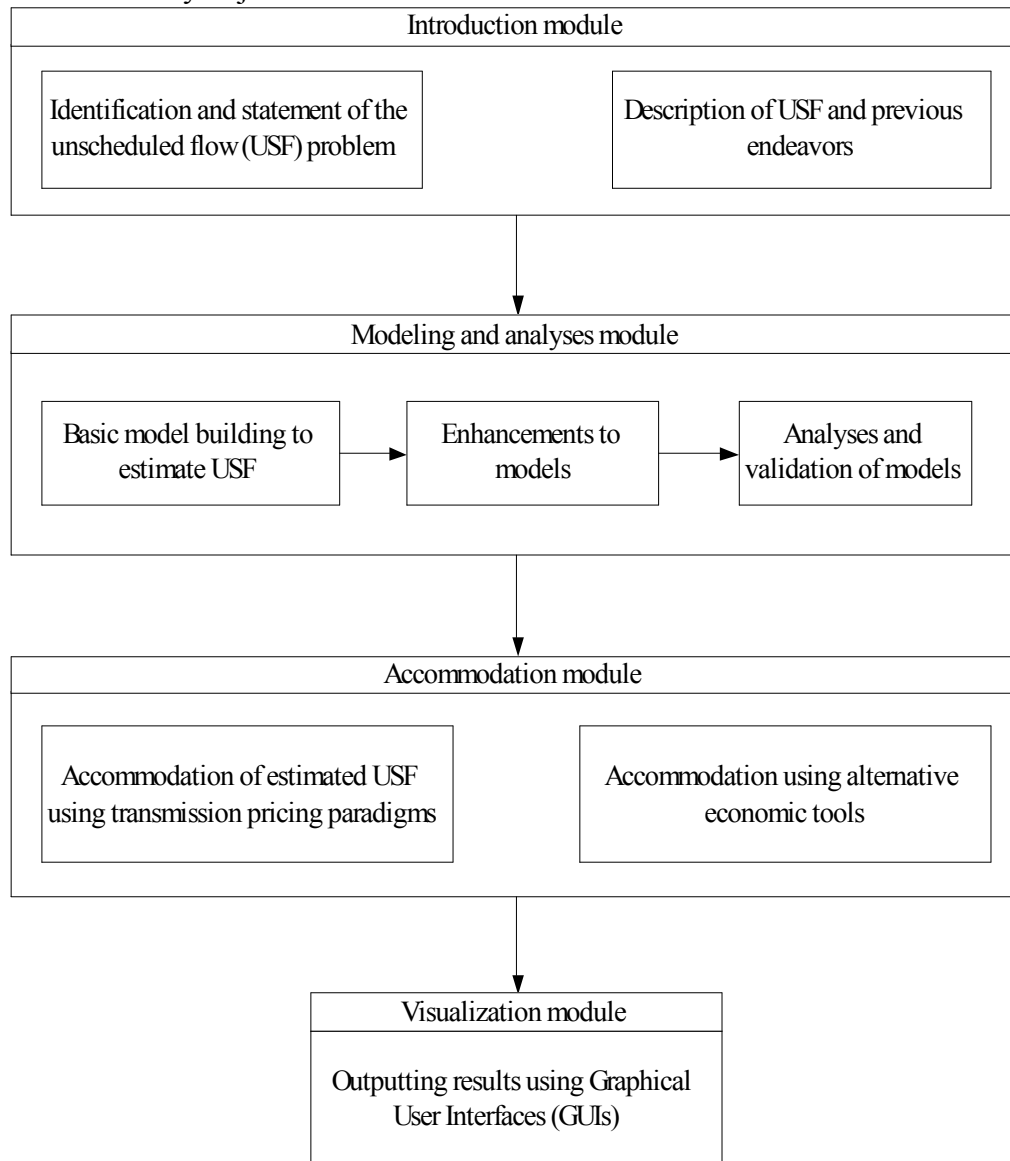


Fig. 1.2 Flowchart of the task modules in loop flow estimation and accommodation

To serve the purpose of creating a comprehensive document on loop flows and to provide references to past endeavors and present practices relating to loop flows, a literature search on the following background materials is performed:

Transmission engineering and electricity markets

- Loop flows, their causes, and effects on wide area systems
- Transmission planning, pricing paradigms, and market structure

Mathematics of estimation

- Matrix computations and the generalized inverses
- State estimation and applications in power systems
- Kalman filtering algorithms
- Advanced linear regression techniques

- Model validation methods

Game theory and applications

Sparse matrix methods of power flow algorithms and  
MATLAB programming and visualization.

### 1.3 The accommodation / control of loop flows

Loop flows have been known to cause serious deviations in power flow from the schedules and the prices cleared for the transactions. To compensate for the losses of the parties not involved in a transaction, there arose a need to estimate the amount of loop flow circulating in the wide area system. In the past, there have been efforts to evaluate parallel flows arising in wide area networks and to assign compensation values to transmission companies for introducing the loop flows into the system. Several techniques have been adopted by the electric industry in the past to estimate the loop flows in a system. However, most of these methods were discarded for the lack of justification to the assumptions made.

The Interutility Transmission Coordination Forum (ITCF) formed the General Agreement on Parallel Paths (GAPP) committee that aimed at calculating the loop flows circulating in a network [1]. The GAPP method used linear sensitivity factors like Power Transfer Distribution Factor (PTDF), Transaction Participation Factor (TPF), and Interface Participation Factor (IPF) [1]. The term PTDF refers to the percentage of actual power transfer flowing on a specific network branch. TPF is the total participation of a system involved in an individual transaction. IPF is a measure of the percentage of power flowing through the interface between neighboring systems. This method had some limitations inherent to it like the application of superposition and linearity to power.

Huang depicted a new method for evaluating and compensating parallel flows in a network that addressed some of the shortcomings of the GAPP method [2]. The method proposed by Huang does not use the linearization concept, is independent of the slack bus, and is transaction based. However, this method works for individual trades and does not deal with compensation or penalty to utilities participating in the loop flow scenario.

The industry practices two other methods of anticipating loop flows. The WSCC training document describes these techniques in detail [4]. One technique is to construct an impedance circle diagram between all nodes in the system from the historical data available. This circle diagram is then used to predict the loop flow for any schedule as a percentage of the actual flow.

The industry also practices the design of a matrix of active power transfers between different utilities for a particular path. This is a numerically cumbersome process and has a disadvantage in addressing only selected lines where the loop flows have historically been observed to occur. The electric power industry also classifies certain transmission circuits as historically loop flow prone and the estimation of loop flows is done selectively on these circuits [4]. All the methods currently in use in the electric industry accommodate the loop flows from a TRANSCO perspective. This implies that the TRANSCOs are charged or compensated for the loop flows in a system.

Chakka, Suryanarayanan, and Heydt described a conceptual state estimation technique for estimating the loop flows in a wide area interconnection [5]. Suryanarayanan, Heydt, Farmer, and Chakka illustrated the technique for accommodating loop flow in an electric power system using a 9 bus test system and a modified Institute of Electrical and Electronics Engineers (IEEE) 30 bus test system [6], [7]. Their work was based on the assumption that the differences in branch flows occurring due to loop flows were linear combinations of the loop flows circulating in the wide area system. Suryanarayanan, Heydt, and Farmer further developed the tech-

nique and introduced a novel formula for accommodating loop flows in a wide area network from a GENCO perspective [8]. Their model was based on minor loop flows in the system and an assumption that major loop flows in systems were linear combinations of the minor loop flows. Suryanarayanan, Montgomery, and Heydt validated the model proposed by the authors of [8] by employing statistical techniques such as normal probability plots and residual plots [9]. This research is based on the same assumptions as [5]-[8] and an extension of the work done by the authors with some enhancements to the estimation techniques and the contribution factor formula described by Suryanarayanan, Heydt, and Farmer in [8]. The technique described in this research shifts the focus of accommodation of loop flows from a TRANSCO to a GENCO perspective.

#### 1.4 Transmission engineering and electricity markets

Transmission engineering and electricity markets is a primary subject of probe toward understanding the physics and engineering of loop flows and the present practices followed by the electric industry to mitigate their effects. The search for literature in this field encompassed the following topics:

- Loop flows, their causes, and effects on wide area systems

Kavicky and Shahidehpour describe the problems arising due to loop flows in transmission systems [1]. This work focuses on defining loop flows and offers a historical account of parallel flows in the United States. This work further defines the working of the GAPP method which relies on reservation, scheduling, and control for addressing loop flows. This research paper also describes the use of TPF and makes some modeling recommendations for alleviating the effects of loop flow.

A committee report by the Current Operational Problems Working Group of IEEE addresses the operating problems that are experienced when dealing with parallel flows in transmission systems [10]. This report defines the parallel flows condition in wide area systems. Separate sections of the report relate to the operating problems associated with parallel flows. These parallel flows occur in different wide area systems in the U.S. like the Pennsylvania- New Jersey- Maryland (PJM) interconnection and the WECC. The concepts of major and minor loop flows are explained and some examples of controlling inadvertent flows arising due to loop flows are also illustrated.

Overbye and Weber define the concept of loop flows briefly and also broach on some topics of controlling the circulating flows. They also delineate the terms PTDF and Transmission-line Loading Relief (TLR) in their work [11].

Falcone describes the problem of loop flows in the wide area connections in the United States [12]. The author refers to a particular example of the power sale involving the American Electric Power Company (AEP), Michigan Energy Coordinating System (MECS), and Ontario Hydro. There is a brief reference to the GAPP committee of ICTF which seeks to solve problems arising from parallel flows by pricing utilities for compensation.

Suryanarayanan, Heydt, and Farmer offer an account of loop flows, their causes and potential adverse effects on wide area systems from both operation and economic perspectives [7], [8]. This work also describes an innovative technique for determining participation for utilities in a loop flow scenario by tagging schedules.

- Transmission planning, pricing paradigms, and market structure

The loop flows caused by utilities are not desired flows and hence compensation is expected to be paid by those utilities responsible. For designing such compensation, in terms of dollars, knowledge of the methods of pricing power is required.

Transmission costing based on cooperative game theories like the nucleolus solution concept and the Shapley value method are discussed in detail by Lo, Lozano, and Gers [13]. The idea of the application of Shapley values to calculate cost is based on the participation of utilities in the system operation. This idea can be extended to assess the cost due of the utility for introducing loop flows into the network.

Shirmohammadi, Filho, Gorenstin, and Pereira explain the various technical issues involved in transmission pricing [14]. They introduce the concepts of wheeling, strategic pricing, and the limiting considerations for transmission pricing. Different transmission pricing models like the rolled-in paradigm, incremental paradigm, and composite embedded/incremental paradigm are dealt. Various transmission pricing methodologies that are in existence such as the allocation pricing methodology and the incremental pricing methodology are discussed. The allocation pricing methodology comprises of the postage stamp method, the contract path methodology, the distance based MW-mile methodology, and the power flow based MW-mile methodology. The incremental pricing methodology types like the Short-Run Incremental Cost (SRIC) pricing, the Long-Run Incremental Cost (LRIC) pricing, the Short-Run Marginal Cost (SRMC) pricing, and the Long-Run Marginal Cost (LRMC) pricing methodologies are explained.

Huang, on designing a new scheme for evaluation and compensation of parallel flows, describes the phenomenon and the effects of loop flows in detail [2]. The work also explains the GAPP method for evaluating loop flows. Huang proposes a slack bus independent and transaction based method for determining and compensating loop flows. This is a pairs-based decomposition method which overcomes the linearity problem of the GAPP method.

An understanding of market structures and the deregulation policies is important in order to design market tools for pricing the USF in wide area networks. The Federal Energy Regulatory Commission (FERC) issued orders 888 and 889 to address open access of transmission circuits and to electronically share information on the ATC of the system through Open Access Same-time Information Systems (OASIS) respectively. A web resource for the FERC orders 888 and 889 appears at [15]. These orders have a direct consequence on markets as they address the issues of deregulation and prevention of monopoly.

Shahidehpour, Yamin, and Li offer a comprehensive explanation of the structure, working, and trading techniques of different types of electric markets in a deregulated scenario [16].

Suryanarayanan, Heydt, and Farmer also describe the different types of markets in a deregulated power system scene and the working of a market [8]. This work describes the importance of pricing loop flows in the market by illustrating an algorithm for a market tool based on state estimation techniques.

### 1.5 Mathematics of estimation

A key feature employed in this research venture is the use of estimation techniques to assess expected loop flows in an interconnected power system. The mathematics of estimation is well documented and numerous good sources of reference are available. From the point of view of this research project, the following topics were considered to be of specific interest.

- Matrix computations and the generalized inverses

A main motivation of this research is to produce computationally efficient approaches for estimating the parallel flows in wide area interconnections. For this purpose, it is intended to study the different methods of solving overdetermined systems with emphasis on matrix computations and the generalized inverses in detail.

Reference [17] deals with linear algebra involving vectors, matrices, subspaces, and Gaussian elimination. This reference addresses the importance of condition numbers in a process, the Least Squares Method (LSM) of estimation, the Singular Value Decomposition (SVD) technique, symmetric and unsymmetric eigenvalue problems, and the constrained LSM. An interesting concept described is the upper bound on the estimate and the residual based on the condition number of the process matrix and the perturbation to the system. A linear equation involving matrices can be solved easily by obtaining the inverse of the process matrix if it is square and nonsingular. However, in most practical applications, the process matrix is overdetermined and hence some techniques for obtaining the generalized inverses are sought.

Campbell and Meyer describe the Moore-Penrose generalized inverse in detail [18]. This book also deals with the linear least squares solutions and the application of the generalized inverse in electrical engineering with references to the  $n$ -port networks and impedance matrices.

Wei describes the analysis and computations involved in the weighted least squares problems and the weighted pseudoinverse problems [19]. Albert details the theory, properties, computations, and the statistical applications of the Moore-Penrose pseudoinverse [20]. Ben-Israel and Greville describe the concepts of achieving the generalized inverse or pseudoinverse of a non-singular matrix and its applications to linear systems [21].

Branham gives basic definitions of floating point numbers, matrices, vector, norms, and condition numbers [22]. It is basically aimed at describing the linear least squares method of solving overdetermined systems using the Moore-Penrose pseudoinverse.

- State estimation and applications in power systems

State estimation, according to Schweppe [23], is the process of estimating the properties of the system or the system state vector using observed data. Even though state estimation by minimization of residuals in a least squares sense was first introduced by Gauss and Legendre in the early 19<sup>th</sup> century, it was not applied to power systems until the 1960s by Schweppe and Wildes. Since then, many researchers have explored different avenues of this application related to power systems. Schweppe describes the various types of estimation techniques applied to power systems and details the subtle differences between state estimation, identification estimation, and adaptive estimation [23]. The work describes the essential steps in system theory viz., modeling, analysis, estimation, and control of process. Schweppe also recommends that there should be ample justification in employing the pseudoinverse based on a least squares sense than just its ease in application. This book also describes stochastic process and the mathematics of white noise.

Heydt describes the basics of state estimation and stochastic modeling in [24]. This book defines the Moore-Penrose pseudoinverse in a least squares sense and devotes an appendix to its mathematics. Also described are the applications of this popular method in power engineering.

Meliopoulos discusses the application of state estimation to power systems and offers a historical note on the progress of state estimation in power engineering. The work delineates the various aspects of state estimation in power engineering, the assumptions made, and the errors

and unreliability of estimation introduced thus. This work is dedicated to discussing the problems arising due to state estimation techniques when applied in mega Regional Transmission Organizations (RTOs) [25].

Wood and Wollenberg introduce the reader to the theme of state estimation in power systems by discussing the basics of matrix formulation, maximum likelihood estimation, weighted least squares method, and several examples of state estimation techniques pertaining to Alternating Current (AC) networks [26].

The mathematics of estimation is explained in [27] by Kreyszig: he deals with vector algebra, linear dependence and independence involved in vectors, inner (dot) and outer (cross) products of vectors. Further, he describes the concepts of matrices and determinants, the arithmetic of matrices, special forms of matrices, the system of linear equations, and the concept of eigenvalues and eigenvectors. He also offers insights into numerical methods in linear algebra and explains the philosophy of LSM. An algorithm for programming the estimator using the LSM is discussed by Späth [28]. Linear regression is discussed in this reference. Optimization and linear programming are some important tools used in state estimation. Heydt deals with the concepts of stochastic methods of power analysis [24]. An appendix is also included on the simplex method in linear programming. Kreyszig introduces the reader to the optimization techniques of linear programming and simplex method and also discusses the theory behind the degeneracy and starting difficulties experienced in simplex programming [27].

The above references were a general overview on the mathematics of state estimation and some generic applications to power engineering. However, there have been researches involving state estimation techniques which have addressed specific needs in power systems as described in the following references. Falcão and de Assis have presented a procedure for the analysis of errors in a linear programming estimator using the weighted least absolute value estimator, applicable to power systems [29]. This work describes the method for identifying bad data during estimation. This work is of importance because of the large amount of data handled in power systems which come intermingled with outliers and bad data.

Holten, Gjelsvik, Aam, Wu, and Liu have compared the various methods of state estimation used in power systems [30]. The focus of their work is to resolve numerical stability problems arising in the gain matrix in state estimations due to ill-conditioned data. Their work compares five different methods for state estimations: normal equations technique, orthogonal transformation method, hybrid technique, normal equations with constraints, and the Hachtel's augmented matrix procedure based on numerical stability, computational efficiency, and implementation complexity. The discussions of Geisler deal with the need for a scaling factor for the gain matrix and a higher precision in factorization with respect to the normal equation with constraints method. The discussions of Boardman on the orthogonal transformation deal with the fact that the method has higher computational memory requirements than other methods but shows numerical stability [30].

Handling of data is a very important feature in state estimation and this has to be done in order to avoid ill-conditioning and redundant data. Filho, de Souza, de Oliveira, and Schilling describe the importance of critical measurements and sets for state estimation [31]. This paper establishes an algorithm for identifying the critical sets and critical measurements. Celik and Abur illustrate the importance of scaling in the weighted least absolute value estimator to avoid the effect of leveraging points or outliers [32]. This work is also helpful in dealing with ill-conditioned data available readily in power system studies. Fig. 1.3 illustrates an example of an outlier in estimation, represented by point *A*.

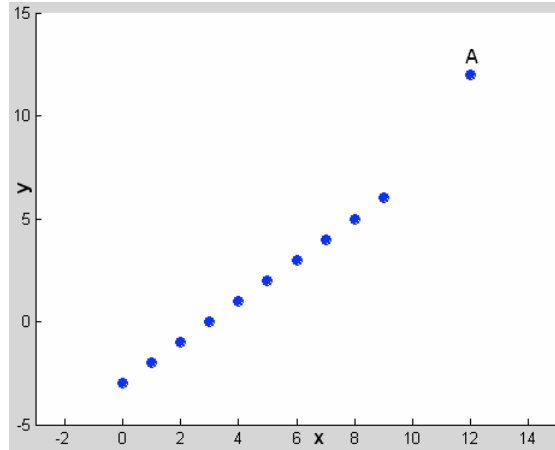


Fig. 1.3 Representation of an outlier in estimation

- Kalman filtering algorithms

Kalman filtering is a dedicated state estimation technique built on the fundamentals of the LSM technique. It is a computer algorithm for the optimal processing of measurements. Since its introduction in 1960, it has been employed in a variety of applications like aided inertial navigation systems, telephone load forecasting, and in power systems relaying. A historically important source of the discrete Kalman filter is the paper written by Kalman in 1960 [33]. In 1961, this algorithm was modified for continuous filtering by Kalman and Bucy [34]. Kalman filtering has applications in state estimation wherever noise contaminated measurements exist. Reference [35] explains the working and some applications of the Kalman filter. Haykin has explained the theory behind the working of the Kalman filter and describes the application of an extended Kalman filter to non-linear systems [36], [37]. Heydt describes the theory of Kalman filters along with other state estimation techniques [38]. Albert also briefly explains the mathematics of the Kalman filters with emphasis on the recursive nature of these filters [20].

Girgis and Brown in [39] introduce the idea of Kalman filtering to computer relaying and describe its application in sensing faults on a three phase line to initiate relaying actions. Girgis has extended the application of the Kalman filter in different avenues of power engineering including tracking harmonics, voltage flicker measurements, and parameter measurements in time varying high frequency transients [40]-[42]. Beides and Heydt have described a Kalman filtering methodology based dynamic state estimation technique of the harmonics present in power systems [45]. Del Castillo and Montgomery have applied a Kalman filter to the development of a control scheme designed toward semiconductor short-run manufacturing [46].

- Advanced linear regression techniques

Most often in practical situations there may exist certain conditions which violate the assumptions of the linear least squares estimation. Non-normality of residuals, presence of near-linear dependencies, multicollinearity among regressors in the incidence matrix, and non-constant variance among residuals are some examples of the violations. These conditions may inflate the variance or change the magnitude and sign of the estimates. In such circumstances, advanced estimation procedures such as robust estimation and biased estimation are to be adopted to avoid unstable estimates.

Of historical importance to robust regression is [54] by Andrews, Bickel, Hampel, Huber, Rogers, and Tukey. This work was one of the first ventures toward documenting the mathematics of robust estimation. All the authors of [54] have contributed to the field of robust regression

by devising special weighting functions that have specific down weighting characteristics. The work was a culmination of a one-year research endeavor carried out at the Princeton University in 1970-71. Other basic references include Andrews [55], Carroll and Rupert [56], Hogg [57], [58], Huber [59], [60] and Rousseeuw and Leroy [61].

Ridge regression is a biased estimation technique used to overcome difficulties in estimation imposed by linear dependencies among variables and non-normal residual distribution. The mathematics of the ridge regression was first introduced by Hoerl and Kennard in the early 1970s [62]. This type of estimation involves the selection of a biasing parameter which indicates a departure from the linear least squares estimation.

Staudte and Sheather describe in detail many topics related to robust estimation including breakdown points, efficiency, asymptotic concepts, M-estimation, and bootstrapping [63]. Gruber makes a comparative study of the different regression estimators that are popularly employed [64]. There is also a section on biased estimators such as the generalized ridge regression estimator and mixed estimators.

Freund and Minton provide a detailed description of linear estimation techniques and basics of multicollinearity [65]. They also describe a ridge estimator for combating multicollinearity among variables in a system. A brief introduction to generalized linear models is also provided. Another biased estimation technique that helps in removing the effects of near-linear dependencies from the results is principal components regression. The method involves the transformation and rejection of the singular values of the incidence matrix, thus eliminating the hazards of a high condition number. Jackson provides a user guide for performing principal components regression [66].

Montgomery, Peck, and Vining give an overview of some of the techniques that can be employed when violations to assumptions are apparent in estimation [67]. The authors describe methods to confront multicollinearity with examples from the practical world of engineering. Popular techniques of biased estimation such as ridge regression and principal components regression are explained. Reference [67] deals with robust regression using weighting functions to down-weight the problematic outliers in estimation.

- Model validation methods

A prerequisite for almost every state estimation or regression problem is a model that describes the process variables. The estimation is performed based on this model and often it is required to test the validity of this model to base confidence on the estimates. Some techniques of model validation are data splitting, application of the knowledge of the process, and checking the model with new data. Montgomery, Peck and Vining describe the techniques of model validation using these techniques [67].

A comparison of the probability plots using least squares and robust estimators is provided by Lawson, Keats, and Montgomery [68]. This work provides a comparison of a variety of robust M-estimators with the least squares estimator and postulates that there is no noticeable difference among the parametric estimates of the least squares estimator and the robust M-estimators. Montgomery and Conard describe the techniques of validation for data from simulation models as compared with original data with a practical application to an actual missile system [69]. Several statistical procedures are described to establish the validity of simulation models.

Suryanarayanan, Montgomery, and Heydt describe a few model validation techniques that pertain to the loop flow problem in wide area interconnected power systems [9]. The



authors describe the validity of the model proposed by Suryanarayanan, Heydt, and Farmer for describing the loop flows in terms of the branch difference flows. The validation is done by employing normal probability plots and residual plots on original and simulated data sets.

### 1.6 Game theory and applications

Game theory is the branch of mathematics that deals with the framework for decision-making in multi-player competition. The application of game theory started with the seminal work of von Neumann and Morgenstern in the 1940s [70]. Game theory gives a mathematical representation to most of the strategies and rational decisions involved in a multi-player game. A concept of a value for each player in the game was introduced by Shapley in 1953 [71]. The so-called Shapley values are the expected marginal contribution of each player in a game based on the order of the coalitions formed in the game. Some other references of game theory and economic theory for topics pertaining to strategic equilibrium, cooperative solutions, large economies, information, knowledge, and utility, game theory for economic analysis and time series analysis include Hart and Neyman [72] and Diestler, Fürst, and Schwödiauer [73].

Brams, Schotter, and Schwödiauer's book serves as a reference on game theory techniques applied to modeling and analyses of voting games, coalitional games, arbitration games, cooperative games, and  $n$ -person games [74]. Lo, Lozano, and Gers describe the application of Shapley values and the nucleolus solution method of profit sharing among transmission companies [13]. The authors compare the above economic theory method with another technique of straightforward profit division called the egalitarian method.

### 1.7 Sparse matrix methods of power flow algorithms

This part of the literature search was limited to the web resource [75] which contains multimedia presentations of Tylavsky on the sparse matrix techniques applied to power flow algorithms. This resource is just one of the many on the internet for computer applications in power flow. One such resource is the web site hosted at [76] by Christie to access a variety of power system test case archives. Most of the test cases used in this research were obtained from [76]. The web site also has links to dynamic data and helps the user in interpreting the power flow data file in IEEE format. Also, Heydt describes in detail the computer algorithms involved in developing an efficient power flow program [24].

### 1.8 MATLAB programming and visualization

A considerable portion of this research endeavor was performed on the software MATLAB. Etter describes some important applications of MATLAB in engineering [77]. The web-site for this software located at [78] and the built-in *help library* in the software are good sources of training in MATLAB.

Visualization of the results obtained by advanced programs is an important issue to be addressed. Mostly, a user may not be involved or interested in the minutiae of an advanced program and may only want to supply the inputs and obtain the results. For this purpose, the development of a user-friendly menu-driven man-machine interface such as a GUI is considered a good practice. MATLAB has a GUI editor, GUIDE, for helping the programmer create end-user GUIs. Two good sources of reference for programming a GUI in MATLAB are Marchand [79] and the built-in *help library* of the software.

## CHAPTER 2

### LOOP FLOWS IN ELECTRIC POWER SYSTEMS

#### 2.1 Deregulation of electric power markets

Electricity deregulation refers to the unbundling of the entities in the electric power industry. Traditionally, the electric power industry consisted of several utilities which were individually responsible for generating electricity, transmitting it to the load centers, and for distributing it to consumers. This vertical structure, shown in Fig. 2.1, was founded on a monopolistic base.

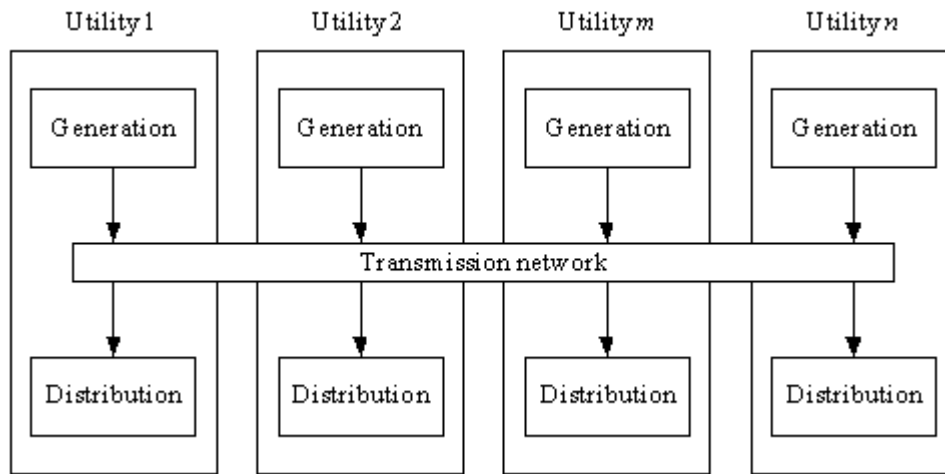


Fig. 2.1 Electric market structure before deregulation

Unbundling is the process of separation of the power generation from the other services. With deregulation came a complete restructuring of the industry. Deregulation introduced new entities in the market scenario which are responsible for either generating power (GENCOs), or transmitting electricity (TRANSCOs), or distributing power to consumers (DISTCOs). This structure, shown in Fig. 2.2, also established the concept of free market competition according to which electricity was traded.

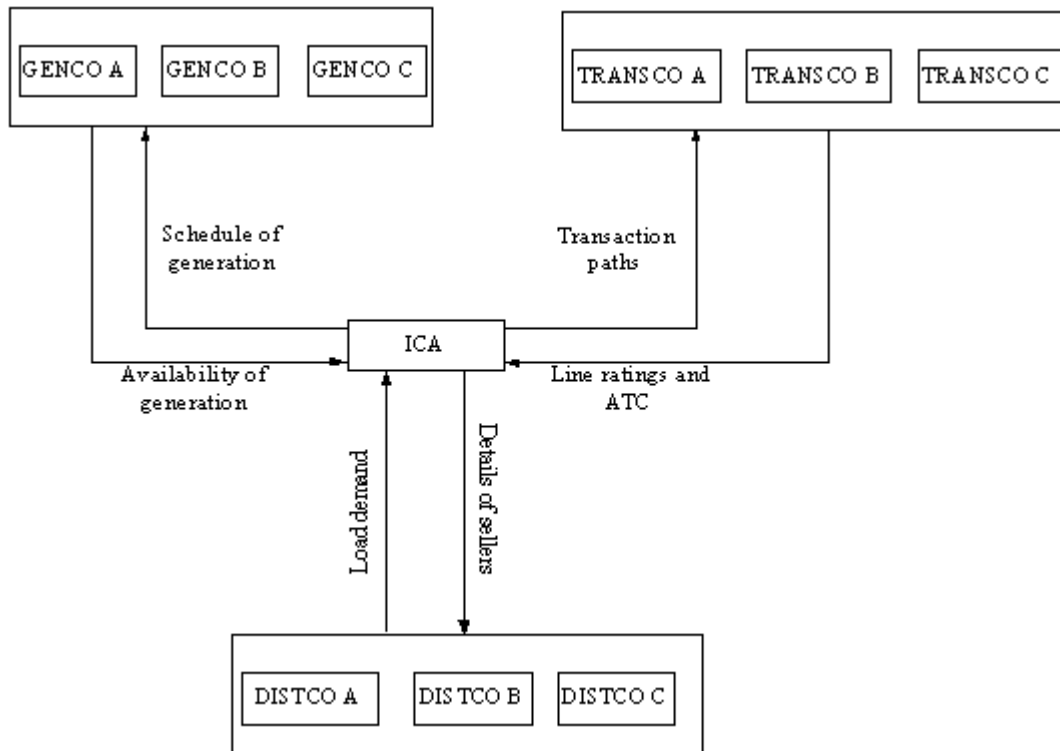


Fig. 2.2 Unbundled electric market structure after deregulation

The deregulation of the electric power market in the U.S. was introduced in the late 1970s to prevent the escalation of a nationwide energy crisis. The National Energy Act of 1978 was passed by the U.S. Congress to conserve energy and increase efficiency by judicious use of resources and amenities by utilities. The National Energy Act included the Public Utilities Regulatory Policies Act (PURPA) which advocated the need for small power productions like cogeneration and renewable energy sources. A direct consequence was the creation of many Independent Power Producers (IPPs) that could participate in the power market competition along with the existing utilities. The FERC was formed from the reorganized Federal Power Commission (FPC) in 1977 [80]. One of the major functions of FERC was to regulate energy transactions and transmission across different states. The FERC was also responsible for other utilities related issues as well.

Under a regulated electric power market structure, the utilities were charging their consumers based on a *rate of return* policy. According to this strategy, the utilities were able to charge the consumers a rate that covered not only the costs incurred in generating, transmitting, and delivering energy but also a fair return of the investment capital [81]. The advantages of an open market with competition among participants were considered a solution to the wide spread energy crisis of the 1970s in the U.S. The deregulation of U.S. electric power market was influenced by the changes brought by the deregulation of other industries such as natural gas, transportation, telecommunication, and airline. Also, the large energy consumers were considered responsible for driving the market toward deregulation in seek of lower energy rates. In compliance with a new electric power market structure, FERC issued the landmark orders 888 and 889 in 1996 [15]. Most of the North American network is slowly deregulating under the guidance of the FERC orders 888 and 889 with mixed results. Some of the operating ISOs in the U.S. are the New York ISO (NYISO), the Midwest ISO (MISO), and the California ISO (CAISO). Fig. 2.3 depicts a timeline graph of some important events in the history of de-

regulation in the U.S. At this point, a brief description of the electricity market of the present day is in order.

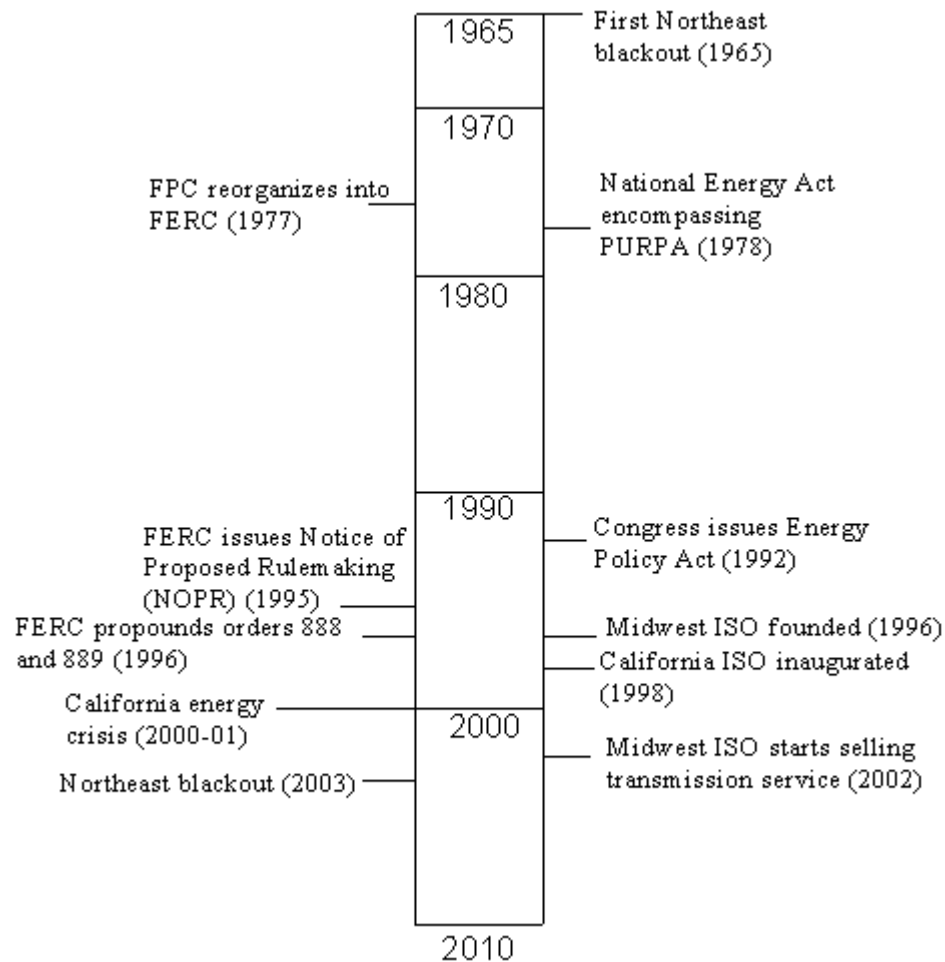


Fig. 2.3 Chronology of events related to electricity deregulation in the U.S.

## 2.2 Electricity markets: types, structure, and operation

Electricity markets have increased dramatically since the attempt to deregulate. Deregulation has been expected to introduce competitive measures to drive down the price of electricity and to reduce the net cost [16]. FERC orders on transmission access were established in order to prevent some utilities from controlling most of the transmission rights in a restructured system. The FERC issued orders 888 and 889 to address open access of transmission circuits and to electronically share information on the ATC of the system through OASIS respectively [15]. This was a major step in the development of the electric power trading market. The FERC order 888 opened the transmission circuits for the use of customers who did not own the circuits at a comparable market price. This order has a direct consequence on the pricing of loop flows if accommodation is employed. Loop flows arising from the participation of a utility may influence changes in ATC on the transmission circuits of another vendor thus causing a need for reiterating the pricing procedure.

Electricity markets in a deregulated scenario are classified based on the type of trading as energy markets, ancillary services markets, or transmissions markets. The electricity market is also categorized depending upon the time of contract as forward market (day-ahead and hour-

ahead) and spot market. Fig. 2.4 shows the classification of electricity markets on the basis of trading type. Markets in electricity are also modeled on many themes such as PoolCo, bilateral contracts, and a hybrid model [8], [16].

In a day-ahead forward market, the bids are generated and schedules are fixed for every hour of the following day. There could be deviations from the schedules set in a day-ahead market and these deviations are balanced in the hour-ahead market. However, in real-time operations of the power system, there exist many special cases which can deviate from the schedules set in the forward market. The real-time market is established to deal with pricing commodities in real-time [8], [16]. Fig. 2.5 shows the classification based on time of scheduling.

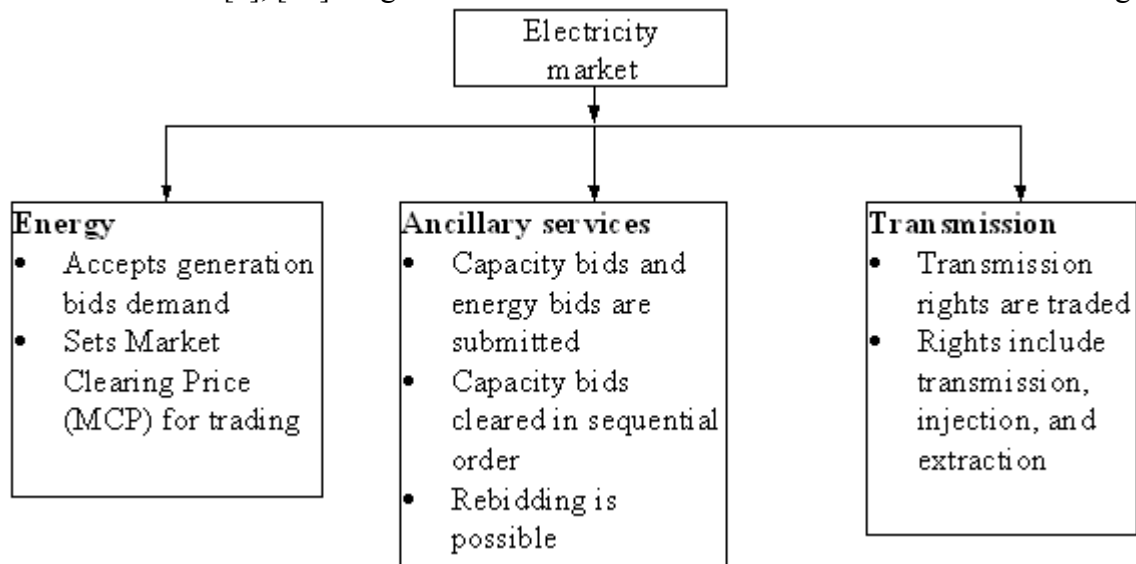


Fig. 2.4 Types of electricity markets based on trading

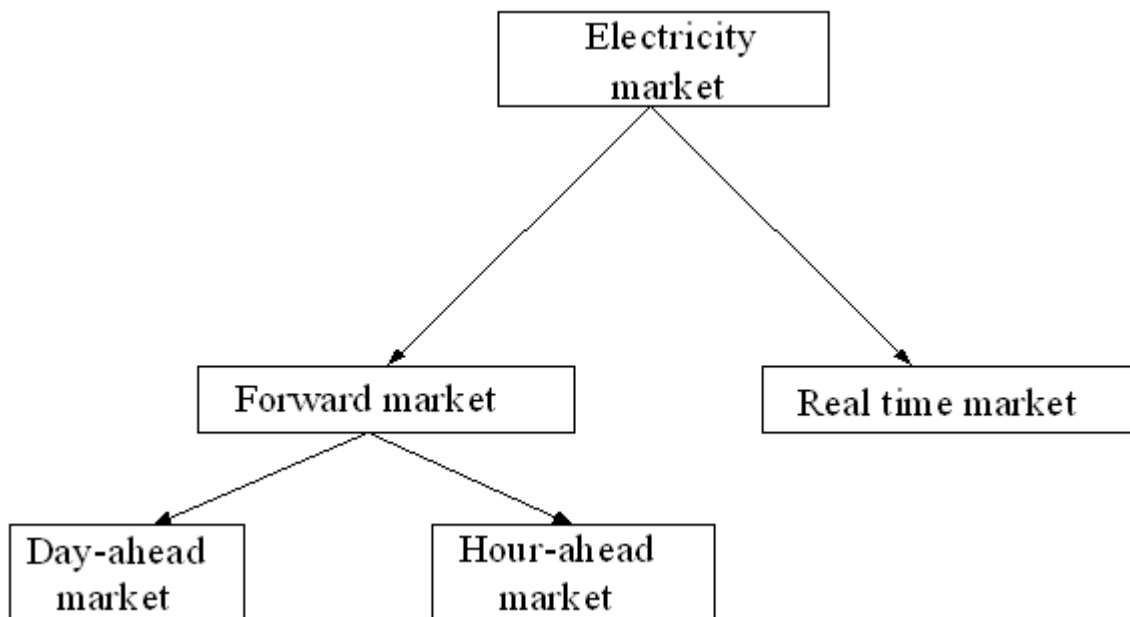


Fig. 2.5 Classification of electricity markets based on time of trading

The key entities in the electric markets are the market operator (ICA or ISO) and the players (GENCOs and TRANSCOs). The other entities include the DISTCOs, customers, aggregators, and brokers. Fig. 2.6 illustrates a typical flow of information in an electricity market.

The generic mode of operation in a market is as follows:

- GENCOs submit bids based on certain bidding strategies to the market after evaluation of resources and load forecasting procedures
- DISTCOs submit load demands to the market based on forecasting, operation, and monitoring of the load profile
- TRANSCOs provide details of line ratings and ATC on the transmission circuits
- The bids and demands are matched based on the market structure and type, and the contract schedules to transmit power are drawn. TRANSCOs are involved in transmission rights issued by the transmission market. Spinning reserves, non-spinning reserves, and replacement reserves are traded in the ancillary services market [8].

The competition driven market for electricity has brought issues related to pricing loop flows in wide area networks. A detailed explanation of loop flows in electric power systems is in order to understand the implications of the loop flows and the design of methods to accommodate them.

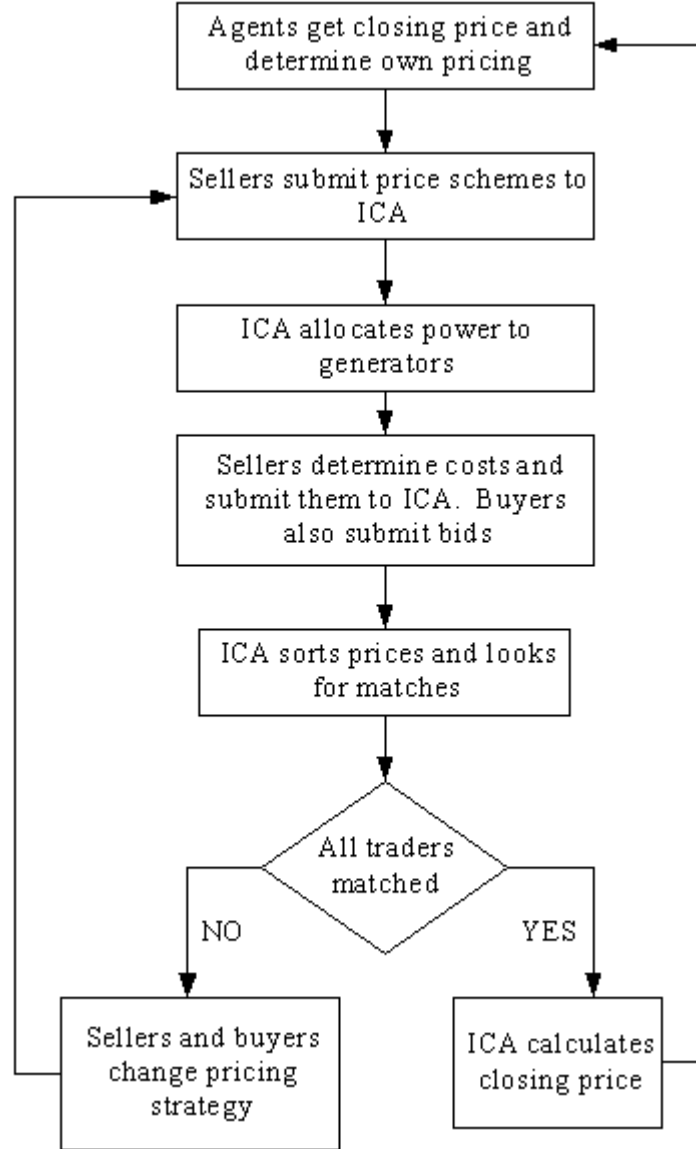


Fig. 2.6 Information flowchart in a typical electricity market

### 2.3 What are loop flows?

Power is scheduled to flow on transmission lines according to transaction schedules between electric power organizations. Typically, a schedule entails a point-to-point transfer of power or energy over a specific transmission path. However, the scheduled power may not flow on the designated transmission paths but may flow on paths determined by physical laws like Kirchhoff's laws and Ohm's law [4], [5]-[9]. This represents a deviation from the desired or scheduled flow. This deviation of the actual power flowing in a circuit from the scheduled flow is termed as *USF* and is assumed to arise due to the phenomenon of *loop flows* [4]-[9], [83]. Loop flows are the flows occurring along a route parallel to the scheduled path [82]. The loop flows are also called *parallel flows* or *circulating flows* and can have adverse effects on the wide area system. Loop flows are an unavoidable phenomenon in wide area interconnected power networks. Mathematically unscheduled flow is expressed as,

$$USF = (Actual\ flow) - (Scheduled\ flow)\ MW. \quad (2.1)$$

Unscheduled flows have been classified as the single most difficult problem of interconnected operations in the WECC history [4]. Loop flows essentially deal with the difference in real power in transmission circuits and not the reactive power. It is common practice in the electric industry for each participating utility to be directly responsible for the reactive power flow associated with their generation. Hence, accommodation of reactive power flow differences in a deregulated network structure is not prevalent.

An example explaining the loop flow phenomenon in an interconnected network is described using Fig. 2.7. An interconnection between three utilities is shown in Fig. 2.7. A power transaction is assumed to be scheduled by an ISO for a transfer of 1000 MW from utility 1 to utility 2 without concerning utility 3. However, when the utility 1 transmits the power to utility 2, 100% of the scheduled power does not follow the path prescribed by the transaction schedule, but a portion flows through the interconnected utility 3. This is due to the well known concept of current division in opposite ratios of the line impedances. The example shows that 270 MW of the 1000 MW transmitted from utility 1 to utility 2 loops through utility 3 while 730 MW of the scheduled power travels to utility 2 via the stipulated transaction path [5]. Table 2.1 describes the desired flow, the actual flow, and the USF occurring in the different branches of the system shown in Fig. 2.7.

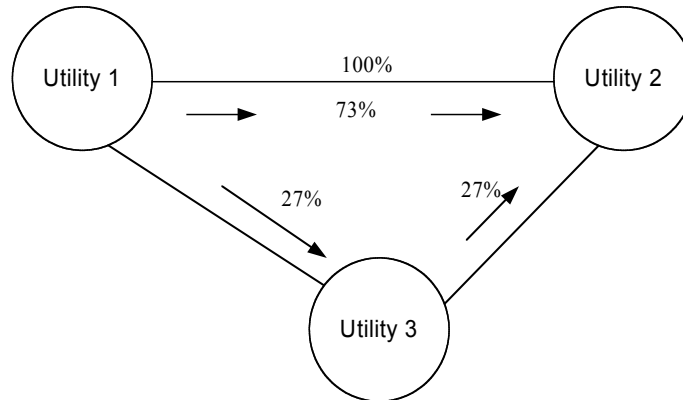


Fig. 2.7 Loop flow in an interconnected power system

TABLE 2.1  
USF in the branches of system shown in Fig. 2.7

| Branch<br>(From – To) | Desired flow (%) | Actual flow (%) | USF (%) |
|-----------------------|------------------|-----------------|---------|
| Utility 1 – Utility 2 | 100              | 73              | -27     |
| Utility 1 – Utility 3 | 0                | 27              | 27      |
| Utility 2 – Utility 3 | 0                | 27              | 27      |

It is observed from Fig. 2.7 and Table 2.1 that there exists a loop flow of 27% of the scheduled power from utility 1 to utility 2, flowing in the counter clockwise direction on the transmission circuits. In a typical wide area interconnection there may exist numerous utilities, transmission paths, and schedules. The increase in the number of players, paths, and schedules also increases the number of loop flows that may circulate in the system. It is important to note that unlike authentic circuit loop and mesh currents that satisfy the Kirchhoff's laws, loop flows are *not* real current flows and do not satisfy the Kirchhoff's laws.

## 2.4 Types of loop flows



Loop flows can be broadly classified into two groups depending on the extent of their flow in an interconnection. The two subdivisions are major and minor loop flows [5]-[10]. Major loop flows are those loop flows which manifest in loops of extensive geographical limits. Minor loop flows are loop flows which occur in comparatively smaller geographical areas. A minor loop flow is also described as a loop flow with the path of circulation restricted to a circuit mesh. According to circuit theory, the number of meshes in a  $B$  branches,  $N$  nodes system is given by

$$\# \text{ of meshes} = B - N + 1. \quad (2.2)$$

Fig. 2.8 depicts the paths of minor and major loop flow types in a typical 6 bus 3 generators 4 loads interconnected system with 8 transmission paths. The three minor loop flows that are restricted to the circuit meshes are indicated by the solid arrow heads and named ' $m1$ ', ' $m2$ ', ' $m3$ '. A major loop flow indicated by dashed arrow head named ' $M1$ ' is shown in the Fig. 2.8. The path of the major loop flow extends beyond a circuit mesh. The major loop flows can be considered a linear combination of the minor loop flows for ease of estimating the loop flows in the system [6]-[9].

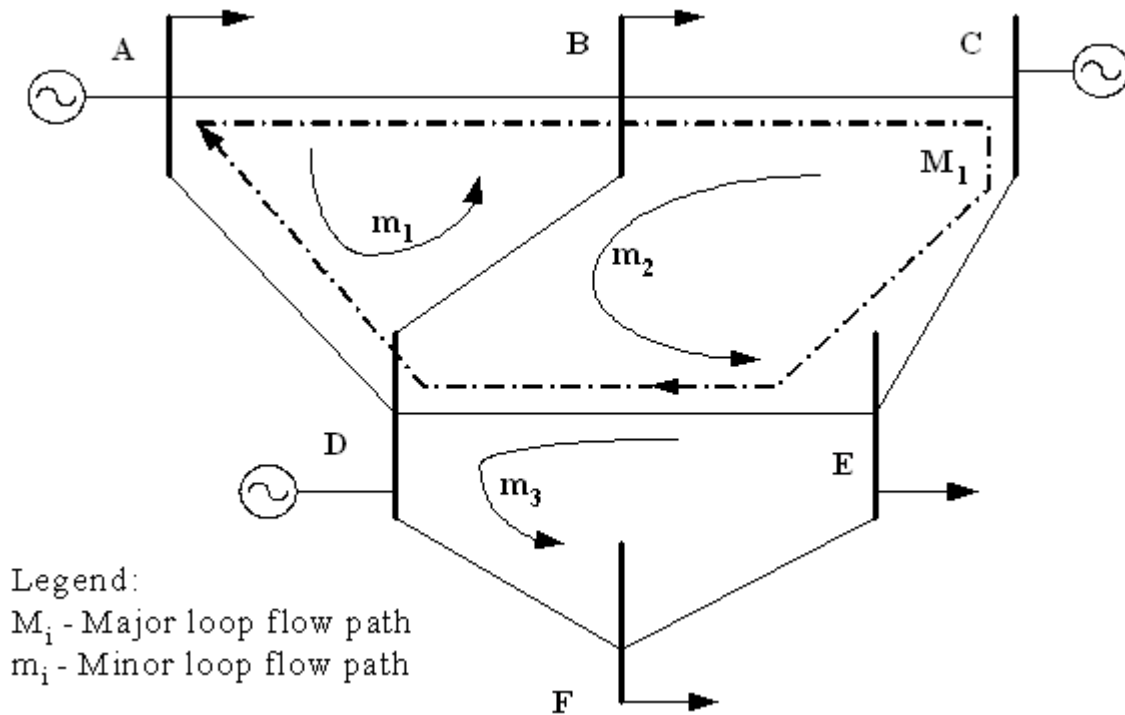


Fig. 2.8 Minor and major loop flows in an interconnected system

Regardless of the type of the loop flow in the system, it is the amount and direction of the loop flow that can be detrimental to the operation of the system. There could be several of these two types of loop flows occurring in an interconnected system and both can have certain potentially undesirable effects on system operation [5]. There are unscheduled flows in the interconnection that do not manifest themselves in loops but exist as individual “difference branch flows”. These are popularly termed as *inadvertent power flows*.

## 2.5 Causes and effects of loop flows

Loop flows are caused primarily because power flow obeys physical laws rather than transaction schedules or legal contract schedules prearranged by an arbitrator. This is compounded by changes in network topologies and other transaction schedules. More system interconnections increase the chances for the occurrence of loop flows. The phenomenon of *loop flows* will arise whenever there exist paths parallel to the scheduled path in the system. The parallel flows in an interconnection have been observed to increase with an increase in transmission distances [5], [6]. This phenomenon causes certain system operating problems as well as potential fiscal effects to electric power entities. The most common adverse effects of loop flows are:

- Limitations to transaction schedules
- Flow path congestion
- Decrease in ATC
- Overloading of lines leading to security and reliability problems
- Forced participation in power transfer
- Market-pricing complications
- Uncompensated losses incurred by a third party [5].

If the loop flows act in a direction such as to increase the power flow in a line far more than the scheduled flow, this may cause congestion of the power flow path. The increase in flow in a line from the desired value also changes the expected ATC of the line. If the loop flows increase the amount of power flowing on a line far above the rated capacity of the line, there could arise unforeseen contingencies such as tripping of lines to avoid damages. This could cause problems related to security of the system and can decrease the reliability of the system [5].

Loop flow causes unscheduled power flow on lines that are not part of a schedule also, thus imposing forced participation of those lines. This can be seen from the example depicted in Fig. 2.7. The scheduled flow is from utility 1 to utility 2 via the line between utility 1 and utility 2 only. However, loop flows force lines between utility 1 and utility 3 and between utility 3 and utility 2 to be involved in the power transfer even when they are not included in the schedule. One method of dealing with loop flows is to compensate the transmission line owner for the inadvertent power flowing on the line and to penalize the owners of the contract path. A fully accepted pricing method is yet to be developed. The loop flows may also increase or decrease the market power of certain players, thus making the market scene shift toward monopoly. Sometimes the losses caused by the inadvertent flows on lines can increase the operating cost to the transmission line owner not involved in the transaction that causes the inadvertent flow [2]. These effects may adversely affect the equitable transmission costs.

## 2.6 Contemporary practices relating to loop flows

Unscheduled flows must be estimated or measured in order to either control or accommodate these flows. The industry has practiced some methods in the past for estimating the unscheduled flows. A popular method used to price the USF in the system involves the design of an incremental matrix for every qualified path in the system. A qualified path is defined as a transmission circuit that has a historical record of at least 100 hours during the last 36 months with path loading of more than 97 percent of the path transfer capability causing curtailment of schedules due to USF [4]. Currently, the WECC system has nine qualified paths in the entire wide area system. Each qualified path has a qualifying direction and a path transfer capability. The present method of ameliorating the loop flow problem is by a combination of accommodation and curtailment by Qualified Controllable Devices (QCD) such as a phase shifter capable of reducing the USF on qualified paths by a specific percentage. The QCD may be owned by an

operator who receives compensation for the use of the QCD by other operators [4]. The procedure followed to mitigate USF in a wide area system like the WECC is as follows:

- Operators are required to accommodate the USF initially by employing their own control devices
- Secondly, coordinated QCD operation may be utilized to accommodate the USF
- Finally, a curtailment procedure for schedules over the qualified path is followed according to an incremental matrix [4].

Another method is a computer program based on construction of a power flow circle diagram which can be used to estimate the expected loop flows in a simple circuit [4], [6]. The power flow circle diagram is marked in percentages of relative impedance going both clockwise and counterclockwise from a reference point as shown in Fig. 2.10. In essence, the consequences of simple current division are used to estimate the current in the circuit, and concomitantly, the power flows are estimated. The power flow circle diagram is an elementary way to obtain an estimate of loop flows in a circuit for a particular contract. The loop flows are estimated according to the following steps from a power circle diagram:

- Locate the starting point, ending point, and the direction (clockwise or counterclockwise) of the schedule
- Calculate the percentage impedance between the extreme points of the schedule
- Calculate the amount of loop flow by multiplying the percent impedance and the schedule amount of power.

A numerical example for this method is provided for ease of understanding. Fig. 2.9 represents an interconnection with generation at points *A*, *B*, *C*, *D*, and *J* and loads at *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, and *J*. A schedule of 1000 MW from point *J* to point *E* in the counterclockwise direction through point *H* is assumed to occur. Using the power flow circle diagram of percent impedances shown in Fig. 2.10, it is computed that the percent impedance between point *J* and point *E* in the counterclockwise direction is about 35%. Hence, the unscheduled flow reaching point *E* from the point *J* in the clockwise direction is obtained by multiplying the relative percent impedance and the scheduled flow. The unscheduled flow in the clockwise direction is found to be 350 MW. A similar example can be conceived for the schedule of 500 MW from point *A* to point *H* in the clockwise direction. The percent impedance is found to be 25% in the clockwise direction. Hence, the unscheduled flow in the counterclockwise direction reaching point *H* from point *A* through points *B*, *C*, *D*, *G*, *J* and *I* amounts to 125 MW. This method is effective in estimating the unscheduled flow in a circuit for a particular schedule but gives no estimate of the amount of unscheduled flow occurring on any specific line. Note that the circle diagram does not account for cases of low  $X/R$  ratios nor cases of high reactive power flows. The circle diagram is not well suited for meshed networks as two distinct paths for a schedule may not be identifiable. The industry also practices a numerically cumbersome process of estimating loop flows by designing a power flow matrix [4]. Hence, the development of an easier and efficient method of estimating loop flows on all transmission paths remains the main motivation of this research work.

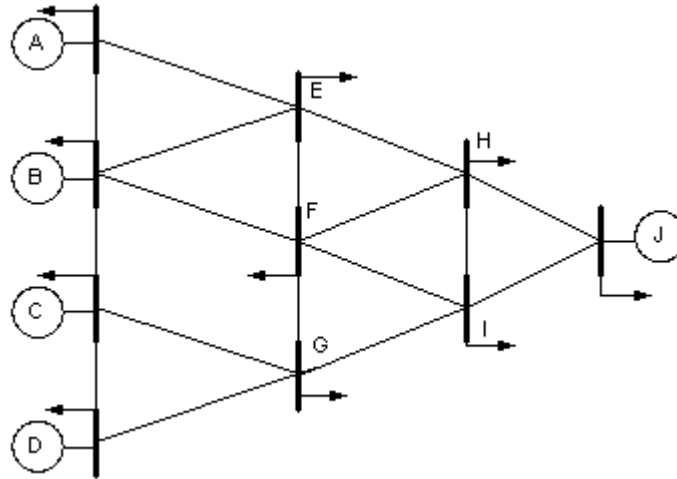


Fig. 2.9 Network with 10 buses and 5 generators

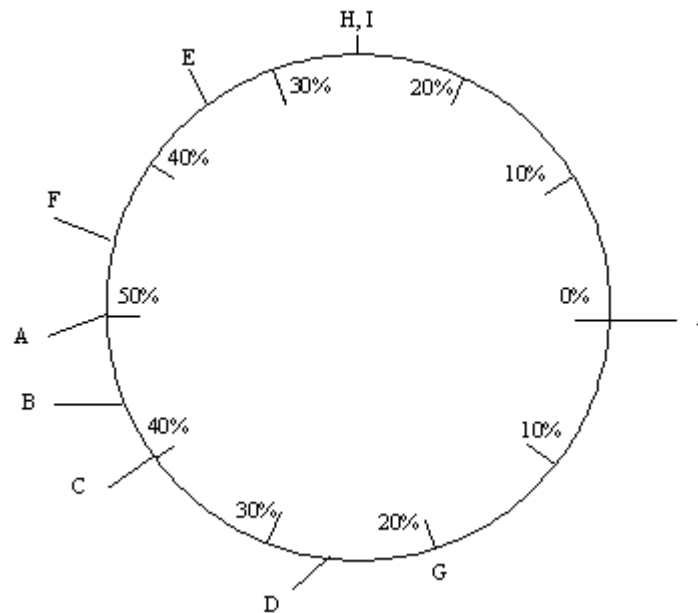


Fig. 2.10 Power flow circle diagram based on percent impedance for network with 10 buses and 5 generators (Fig. 2.9)

All the methods addresses USF over selected qualified paths and do not involve the utilities. Also, the issue of deviation of prices due to USF that do not cause an operational constraint in the system are not addressed. Hence, a method that is capable of addressing USF issues over all the transmission circuits and for charging or compensating the utilities for thrusting USF in a system is required.

There have been efforts in the past relevant to two aspects of confronting loop flows in interconnected systems: controlling loop flows in a systems perspective and accommodating loop flows in a markets perspective.

#### *Techniques to control loop flows*

Unscheduled flows are best controlled by the following methods:

- By employing phase shifting transformers [4]

- Varying reactance of transmission lines [4]
- Utilization of FACTS devices [5], [83]
- Installing flow limiting network elements at appropriate locations in the system [47].

Phase shifting transformers are the most popular way in which loop flows are controlled. These transformers have phase shifters which alter the phase angle  $\delta$ , and hence control the power flow according to the power-angle equation,

$$P_2 = |V_2|^2 G_{22} + \frac{|V_1||V_2|}{X} \sin(\delta - \gamma), \quad (2.3)$$

where  $P_2$  is the real power delivered to the receiving bus,  $|V_1|$  is the magnitude of the voltage at the sending bus,  $|V_2|$  is the magnitude of the voltage at the receiving bus,  $\delta$  is the difference between the voltage angles at the terminals,  $G_{22}$  is the self-conductance of the receiving bus,  $\gamma$  is the argument associated with the mutual admittance, and  $X$  is the line reactance. Equation (2.3) can be simplified for practical purposes by considering a lossless line with negligible values of  $G_{22}$  and  $\gamma$  such that the power sent is equal to the power received [4]. The power angle equation, (2.3) reduces to the lossless form,

$$P_2 = \frac{|V_1||V_2|}{X} \sin(\delta). \quad (2.4)$$

Phase shifting transformers control the power flow by changing the value of  $\delta$  and thus make the power flowing in the circuit as desired. Several phase shifters in the western U.S. are installed to cause the actual flow on a path to equal the scheduled flow [4]. Varying the reactance can change the amount of power flowing in a transmission line as can be seen from (2.3). Switched series capacitors offer an alternative for this purpose. FACTS devices such as the Unified Power Flow Controller (UPFC) can be connected at strategic positions in the wide area network for controlling loop flows, as these devices are capable of altering the equivalent line reactance [5], [36], [83]. Perhaps, the major disadvantage of employing FACTS controllers is the cost/benefit ratio of this technology: Power flow control alone does not take advantage of the high speed electronic switching of FACTS controllers. Flowgates are network elements that limit the flow of power in any transmission line [5], [47]. They can be physical devices or financial constraints that do not allow any more power in a transmission line than a certain fixed amount and can be used to prevent overloading problems arising due to unrestrained loop flows [5], [47].

The mentioned methods of controlling loop flows may be efficient but have the disadvantage of being cost prohibitive. Installing control devices involves capital investment by only certain players in the electric power market, thus distorting the deregulated structure of the market by making selected players more powerful than others. Hence, a method of equitable accommodation of loop flows instead of controlling them is sought [4].

#### *Techniques to accommodate loop flows*

There have been some efforts to accommodate loop flows in interconnected systems by employing specific pricing techniques. The best known of these methods are the following:

- The GAPP method
- Transaction pairs-based decomposition method [2].

The GAPP method is a technique that involves the use of the PTDF, TPF and IPF. The PTDF refers to a measure of the power flow in any branch of a network, generally expressed as a percentage of the total power flow. The TPF gives the participation of a system in an actual transaction. The IPF is the percentage of power flow in any interface between two adjacent interconnections. The GAPP technique is slack bus dependent, ignores the effect of reactive power on generators in the system, and does not consider interaction among simultaneous trades. The transaction pairs-based decomposition method overcomes some limitations of the GAPP method. The transaction pairs-based decomposition method has higher computational efficiency and is easier to use than the GAPP method [2], [5]. The transactions pair-based method works for individual trades and does not deal with compensation or penalty to utilities participating in the loop flow scenario.

There is a need for a method of accommodating loop flows, pricing participating utilities and rewarding the forced participants. This research work is aimed toward identifying such a technique.

## CHAPTER 3

### MODEL BASED ESTIMATION OF LOOP FLOWS

#### 3.1 An introduction to state estimation models

State estimation is a specific type of estimation theory which uses physical models and measurements to predict the states of a system. In state estimation, the states of a system under new conditions are predicted using the knowledge of the mathematical model of the process, past and present states of the system, and the observables. In effect, state estimation is the deviation of a mathematical model from historical measurements. The technique of state estimation can be used when the system possesses bad data, as bad data rejection is a common practice preceding estimation. State estimators can accommodate redundant measurements and models. Estimation involves the minimization of differences between the measurements and a physical model. The procedure of estimation can be carried out by a state estimator, which maybe either a static or a dynamic device, that uses the measurements given to it as inputs and gives the estimated states of the system.

The basic linear scalar state estimation problem can be described as,

$$Hx = z, \quad (3.1)$$

where  $H$  is the mathematical model of the process or the process matrix,  $z$  is the observable or measurement of the system (output), and  $x$  is the state of the system which is estimated. The linear scalar equation can be transcribed to a vector scale by considering the matrix equivalent of (3.1) in which  $H$  is an incidence or process matrix describing the process,  $z$  is a column vector of observables, and  $x$  is a column vector of all the system states that are to be estimated. Equation (3.1) is a model for estimating the states of a system from observables and process knowledge.

The system of equations has a unique solution if matrix  $H$  is square and nonsingular. If matrix  $H$  has more columns than rows, this represents an underdetermined system where the number of unknowns to be estimated is more than the number of equations. The vector  $x$  has infinitely many solutions in this case and the method of solving such a system of equations is by assuming that the unknowns that are not considered in the solution take practical values [24]. The interesting and practical case is encountered when the matrix  $H$  is rectangular with more rows than columns. This represents an overdetermined system of equations with more equations than the number of unknowns. The system of equations in this case is solved to yield a best-fit estimate of the states of the system,  $\hat{x}$  by a minimization of a function of the error  $e$ , such that,

$$e = |\hat{x} - x|, \quad (3.2)$$

where  $x$  is the true state of the system.

#### 3.2 Model adequacy

A design model is always subjected to model adequacy tests and validation prior to use in practical situations. The model adequacy tests are performed to determine if the model fits the data to some accuracy. Methods such as analysis of residual plots and normal probability plots are effective tools to conclude adequacy of a state estimator.

The success of a good estimation is defined by the extent of the error between the true values and the estimated values of the state variables – the lower the error, the better the estimate. The exact error cannot be calculated, as the true state of the system is not usually known. However, with the knowledge of the right side of (3.1), the residual  $r$  can be calculated,

$$r = |H\hat{x} - z|. \quad (3.3)$$

The adequacy of a model can be established by plotting the normal probability plot of the residuals. If the residuals appear along a straight line in this plot without serious deviations in the tails, a fair amount of adequacy to the model can be assumed. This is in conjunction with an initial assumption to the process of estimation: errors are independent and normally distributed.

Plotting the residuals against the estimated states of the system can also be used to verify model adequacy. The residual plots are expected to be a random scatter without any discernible pattern. The lack of a pattern to the plot indicates that there may be a desired non constant variance among the errors. The plots may also reveal patterns indicative of a non linear relationship between the states and the observables unlike (3.1). In such a case, remodeling of the relationship is required. Hence, these plots serve as useful tools for establishing the adequacy of a mathematical model.

A linear model describing the relationship between the minor loop flows, the deviation of power flowing on transmission circuits, and their paths can be established. The following sections describe the conceptualization of the model, estimation of loop flows, and verification of adequacy of the model.

### 3.3 Conceptualization of the loop flow problem

The loop flows that occur in wide area interconnected systems are problematic and may require curtailment using control or accommodation by suitable pricing methodologies. For these purposes, an estimate of the loop flows in the system is desired. A linear method of estimating the loop flows is conceptualized as described below. The loop flows in reality are manifested as a mathematical difference between the scheduled flow and the actual power flow on a transmission line. This difference on a branch between the scheduled and actual flow is termed *difference branch flow* [5], [6].

The number of meshes  $L$ , from a network perspective, is given by

$$L = B - N + 1, \quad (3.4)$$

where  $B$  is the number of branches and  $N$  is the number of nodes in the system. The estimation of the loop flows is done under the following assumptions. The difference branch flows are assumed to be linear combinations of the minor loop flows in the system. It is also assumed that only minor loop flows are of immediate concern and that major loop flows can be obtained by the linear combination of the minor loop flows. Even though, the minor loop flows that are estimated do not represent the states of a system, the state estimation technique can be employed with justification that the branch difference flows are manifested as linear combinations of the minor loop flows in a system [5]. A linear model relating the magnitude and path of the minor loop flows to the branch difference flows is assumed. The linear overdetermined system of equations is of the same form as (3.1),

$$Hx = z,$$

where the order of the process matrix  $H$  is  $(B \times n)$ ,  $B$  represents the number of branches in the system and  $n$  represents the number of minor loop flows in the system. A column vector  $(B \times 1)$  of the difference branch flows forms the measurement (observation) vector  $z$ . This overdetermined system of linear equations can now be solved using popular techniques of state estimation to obtain a best fit of the column vector  $x$   $(n \times 1)$ , which consists of the estimates of the minor loop flows in the system.

There are two distinct methods for setting up the incidence matrix  $H$  for the estimation problem. The first method is done by assuming the loop flows to circulate in meshes between the nodes of the system alone. In the second approach, the occurrence of the loop flows that circulate



in meshes between the nodes and in loops existing between a fictitious ground node and every node of the system, called ground flows, is assumed. In the latter study, the total number of branches in the system is given by the sum of the number of loop flows and the ground flows. The former method is proposed for efficient numerical estimation of the loop flows as the accommodation of the large number of ground flows may make the process cumbersome. Also, the decision to neglect the ground flows in setting up the process matrix is validated by the fact that the flows considered for the estimation problem are the difference flows and not the actual flows themselves.

A reason for concern in setting up the incidence matrix may be attributed to the heuristic method of choosing the path of the minor flows in a wide area system. It may be argued that the choice of minor flows may differ from one setting to another depending upon the user. However, the process matrix may be set up using the network flow graph to identify the path of the minor loop flows and assigning the appropriate elements to the incidence matrix. This method may be corroborated by the phenomenon of Occam's razor [51]. According to the Occam's razor principle, in any juncture of choice in a scientific exploration, the choice with the least assumptions introduces the minimum error, inconsistency, and redundancy. Occam's razor is also termed the principle of parsimony and is widely used in statistical and technical decision making. An infinite number of models can be proposed to explain a given set of observations by considering the observed set of data as a subset of an infinite observation set. By adopting the Occam's razor, the model with the least assumptions and consequently the least error can be chosen from the infinite set of models. The principle of parsimony is also adopted in mathematical modeling of systems and is termed the principle of uncertainty maximization, wherein the model that minimizes the number of additional assumptions is chosen over the rest [51], [52]. In the loop flow problem, the process matrix is designed with the minimum number of minor flows in the system according to the network graph as opposed to the assumption of more flows in the system. The justification for using only the minor flows follows from the fact that the variance of the estimates (minor loop flows) may be inflated due to the addition of more regressors in the model. Hence, in the design of a process matrix for the loop flow problem, the method involving only the minor loop flows is chosen over other complicated methods that incorporate additional assumptions regarding the ground flows [9].

Prior to the estimation of minor loop flows in a system, it is intended to establish a statistical level of confidence in estimation based on the structure of the process matrix  $H$ . For this purpose, a unique statistical test based on eigen analysis, Variance Inflation Factors (VIFs), rank, and the Willan-Watts test is devised. In this test, the eigenvalues of the matrix  $(H^TH)^{-1}$  in correlation form are calculated. The condition number of the  $(H^TH)^{-1}$  matrix is determined as the ratio of the maximum eigenvalue to the minimum eigenvalue. The condition number of  $(H^TH)^{-1}$  is a threshold diagnostic for detecting multicollinearity among the columns of  $H$  and is conservatively fixed as 100. Variance inflation factors are defined as the diagonal elements of the  $(H^TH)^{-1}$  matrix in correlation form. The VIFs for each term in the model is an indicator for the increase in the confidence interval of the estimate due to multicollinearity. Large VIFs are indicative of the presence of near-linear dependencies among the columns of  $H$ . A practical consideration in statistics encourages the threshold value of VIFs to be fixed at 5. The rank of the  $(H^TH)^{-1}$  matrix is determined and if the rank is smaller than the number of estimates or the columns in  $H$ , a rank deficient problem indicating linear dependencies among columns of  $H$  is present. Willan-Watts test is a popular multicollinearity diagnostic that measures the percentage increase in the volume

of the joint confidence region for the estimates resulting from near linear dependencies in  $H$ . The test statistic for the Willan-Watts is given by,

$$\left(\sqrt{|H^T H|} - 1\right), \quad (3.5)$$

A conservative threshold for detecting multicollinearity using the Willan-Watts test statistic is fixed at 50%. Boolean flags are set according to the results of each of the individual tests. A value of condition number greater than 100, any VIF greater than 5, a rank deficiency, or a Willan-Watts test statistics greater than 50% operates a Boolean flag to be set to 1. A level of statistical confidence in estimation is determined depending upon the values assigned to the Boolean flags as depicted in Table 3.1.

A high level of statistical confidence in the estimation guarantees a well structured process matrix  $H$  devoid of problems relating to multicollinearity and exact linear dependencies among columns. A low level of statistical confidence in estimation may prompt a redesign of the  $H$  matrix. Since the test is stringently conceived, a medium level of confidence in estimation need not be a cause for concern in estimation; however, remodeling the process matrix or re-specification of the model may be considered as an alternative.

TABLE 3.1  
Boolean flags for statistical confidence of estimation

| Statistical quantity        | Criterion for Flag = 0              |              | Criterion for Flag = 1              |  |
|-----------------------------|-------------------------------------|--------------|-------------------------------------|--|
| VIFs                        | Max(VIF) < 5                        |              | Max(VIF) ≥ 5                        |  |
| Rank                        | Rank (H) = Number of variables      |              | Rank (H) < Number of variables      |  |
| Eigen analysis              | Cond (H) < 100                      |              | Cond (H) ≥ 100                      |  |
| Willan-Watts test           | Δ(Confidence Interval length) < 50% |              | Δ(Confidence Interval length) ≥ 50% |  |
| Confidence level indicators |                                     |              |                                     |  |
| Statistical confidence      | HIGH                                | NOT HIGH     | LOW                                 |  |
| Criterion                   | All Flags = 0                       | Any Flag = 1 | All Flags = 1                       |  |

### 3.4 Methods of estimation

The loop flows in a wide area system may be estimated using a variety of techniques of state estimation by solving (3.1). The methods adopted for estimation may be based on the  $L_1$ ,  $L_2$ , and  $L_\infty$  norms. In the case of loop flows estimation described above, several methods of minimization based on  $L_p$  norms were carried out and it was established that the least squares ( $L_2$  norm) based method was the least error inducing, conditionally fit, and numerically faster to use than other  $L_p$  norm based methods [6], [7], [8]. Some of the  $L_2$  norm based estimation methods and related estimation techniques are described.

#### *Least squares method*

In most practical cases, the process matrix  $H$  may represent an overdetermined system with rectangular structure of more rows than columns. In such cases, estimation is performed by least squares techniques of error minimization. A popular method of estimation based on the least squares minimization is the Moore-Penrose generalized inverse or the pseudoinverse. The pseudoinverse of a matrix  $H$ , denoted as  $H^+$ , is a unique matrix satisfying certain properties [17]-[21]. The pseudoinverse reduces to the common inverse if matrix  $H$  is square and nonsingular. In the USF problem, the estimates can be derived as

$$\hat{x} = (H^T H)^{-1} H^T z, \quad (3.6)$$

where  $H^T$  refers to the transpose of matrix  $H$  and  $(H^T H)^{-1} H^T$  corresponds to a generalized of the incidence matrix  $H$  in a least squares sense.

#### *Kalman filter*

Oftentimes, the observables or measurements in a system are contaminated with noise. This may arise due to several reasons like measurement instrumentation error, operator induced error, and telemetry induced error. Using the noisy data for estimation of the states might yield erroneous results which may potentially harm the operation of the system. Hence, there is a need to filter the measurements to obtain a reasonably noise-free data set, which can be employed in estimating the states of a system. A method to recover data from noisy measurement is by recursive estimation. The pseudoinverse technique can also be employed to obtain the best unbiased estimates in cases of Gaussian noise contamination associated with the measurement vector  $z$ . However, the method may not produce the best estimates when the type of noise contamination in observables is non-Gaussian. For estimation of the loop flows in such circumstances, it is desired to design a discrete Kalman filter (KF). A brief introduction to signal processing terms and a detailed description of the working of the discrete KF algorithm is provided in Appendix A.

Knowledge of the mathematical model of the physical process is a requirement for effective recursive estimation. However, most real-life processes cannot be modeled perfectly and hence an approximate process that describes most physical processes is required. A *Gauss-Markov* (GM) random process is one such approximate process which can be employed to fit most physical processes with a reasonable degree of accuracy [35]. A GM random process is defined as a Gaussian process,  $X(t)$ , with exponential Autocorrelation Function (ACF). The ACF of a random process is a measure of how well correlated a process is with itself at two different time levels.

Equation (3.7) depicts the ACF of a GM random process  $X(t)$ ,

$$R_X(\tau) = \sigma^2 e^{-\beta|\tau|}, \quad (3.7)$$

where  $R_X(\tau)$  is the ACF,  $\sigma^2$  is the mean-square value of the process,  $\beta^{-1}$  is the time constant, and  $\tau$  is the time difference variable of the GM process,  $X(t)$ . The structure of the GM process is noise-like and the exponential nature of the ACF forces the ACF to zero when  $\tau$  approaches infinity. The GM process has simple mathematical expressions and can be used to approximate a mathematical model for many physical processes with considerable accuracy [35].

The measurement vector in the loop flow problem is obtained as discrete observations of the deviation of actual flows from the scheduled flows occurring on transmission circuits. A discrete Kalman filter may be employed for estimating the loop flows in such cases. The discrete KF algorithm handles discrete measurement data which are contaminated with noise. This method of recursive estimation predicts the states of the system by processing noisy data and an a priori estimate of the states of the system.

In most practical cases, not only does the measurement vector  $z$  come contaminated with noise but also the  $H$  matrix may possess structural imperfections such as linear dependencies. Estimation using the unbiased LSM technique may not provide the best estimates in such circumstances. Advanced estimation procedures that depart from the LSM method and introduce a bias in estimation may be employed to overcome difficulties imposed by multicollinearity.

Some of the methods include ridge estimation, robust estimation, and principal components analysis.

### *Ridge estimation*

The process matrix in many practical situations may possess near linear dependencies or multicollinearity effects among the regressors. A direct result of this is rank deficiency and associated problems of untrue estimates. A popular method to overcome the obstacles of estimation with a multicollinear  $H$  matrix is ridge estimation. The ridge estimation technique is a departure from the linear LSM. The LSM is often called the Best Linear Unbiased Estimator (BLUE) which produces estimates with minimum variance among all other unbiased estimators. However, if multicollinearity in  $H$  is profound, the estimates obtained using BLUE may be large, even though the variance may be the minimum among all other unbiased estimators. Hence, it is required to introduce a biasing quantity in the estimation procedure. Ridge estimation modifies the normal equations of the LSM by the addition of a non negative biasing parameter,  $k$ . The introduction of the biasing parameter reduces the variance and consequently decreases the confidence interval lengths of the estimates. The ridge estimator is given as,

$$\hat{x}_r = (H^T H + kI)^{-1} H^T z, \quad (3.8)$$

where,  $k \geq 0$  is the biasing parameter and  $I$  is the identity matrix. The ridge estimator reduces to a linear LSM when  $k$  goes to 0. The bias in estimation increases and the variance of the estimates decreases as  $k$  increases. Hence, a judicious value of  $k$  is required to be used. The choice of selecting a value of  $k$  is dependent on the analyst and the problem at hand. There are several methods to select a value for the biasing parameter and some of the methods are described in detail in [67]. A popular method for choosing appropriate values of the biasing parameter may be determined by an inspection of a ridge trace. A ridge trace refers to the plot of the elements of the  $\hat{x}_r$  vector versus  $k$  for values of  $k$  ranging in the interval  $[0, 1]$  [67]. The plot reveals severe multicollinearity problems by depicting the instability among the estimates; the values of the estimates may vary dramatically as the value of  $k$  increases. The value of  $k$  at which the ridge estimates stabilize may be used as an appropriate biasing parameter. It is however desired to select a small value of  $k$ , so that there is a compromise between the extent of bias and the decrease in variance of the estimates [67].

### *Robust estimation*

Sometimes, data may contain outliers, highly influential points, and may follow a non-normal distribution which may possess the capacity to distort estimates. The nonnormality of data may be determined by studying the normal probability plots and looking for tails in the distribution. Outliers may be detected by residual plots. Outliers may fall under one of the following categories:

- Regression outlier
- Residual outlier
- X-space outlier
- Y-space outlier and
- X- and y- space outlier.

A regression outlier refers to a deviant point from the linear relationship determined by the remaining points. A residual outlier denotes a point possessing a large value of standardized residual. An  $x$ -space outlier and a  $y$ -space outlier refer to a point having one or more remote  $x$

and  $y$  coordinates respectively. An  $x$ - and  $y$ -space outlier represents a data point that has remote  $x$  and  $y$  coordinates [67]. Fig. 3.1 illustrates the different types of regression outliers.

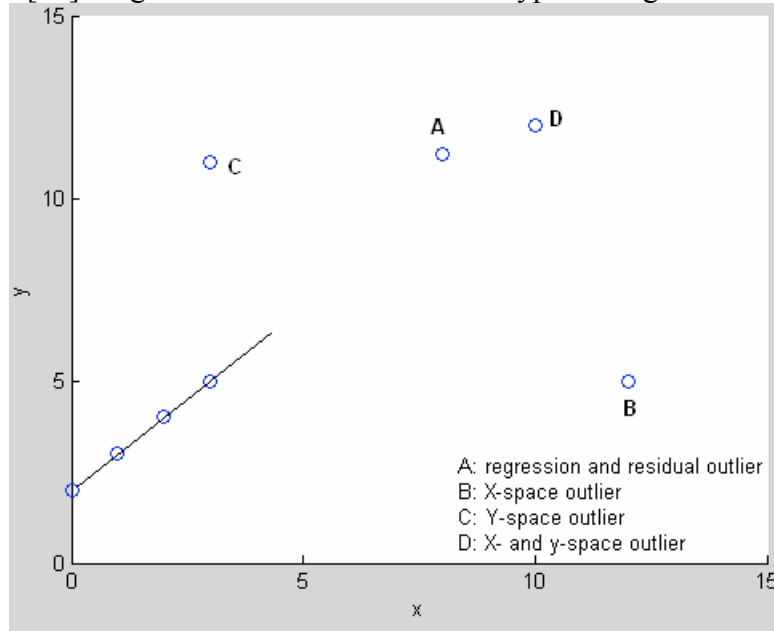


Fig. 3.1 Types of outliers in regression.

The presence of nonnormally distributed data and/or outliers may yield poor estimates when LSM (BLUE) estimators are used. Robust estimation is a procedure that dampens the highly influential effect of observations when LSM is used by leaving out large residuals associated with outliers [67]. A robust estimation technique should essentially yield the same estimates as that of LSM when normally distributed, outlier free data is used. Robust estimators or M-estimators are a class of maximum likelihood estimators that assume an appropriate choice of distribution for the errors. LSM is a special type of an M-estimator that assumes normal distribution of errors. M-estimators have a likelihood function  $p(e)$ , an influence function  $\psi(e)$ , a range, and a tuning constant  $t$ . Table 3.2 lists the different functions and range of some of the commonly employed M-estimators.

TABLE 3.2  
Robust criterion functions for m-estimators [67]

| Criterion  | $p(e)$                                | $\psi(e)$          | Range           |
|--|---------------------------------------|--------------------|-----------------|
| Least squares  | $0.5e^2$                              | $e$                | $ e  < \infty$  |
| Huber's $t$ function                                     | $0.5e^2$                              | $e$                | $ e  \leq t$    |
| Ramsay's $E_a$ function<br>$a = 0.3$                     | $a^{-2}\{1 - \exp(-a e )(1 + a e )\}$ | $e\{\exp(-a e )\}$ | $ e  < \infty$  |
| Andrews' wave function<br>$a = 1.339$                    | $a[1 - \cos(e/a)]$                    | $\sin(e/a)$        | $ e  \leq a\pi$ |
| Hampel's 17A function<br>$a = 1.7, 3.4, \text{ or } 8.5$ | $0.5e^2$                              | $e$                | $ e  \leq a$    |

The least squares M-estimator has a  $\psi(e)$  function which is unbounded. This may make the estimates non-robust when data from a heavy tailed distribution is used [67]. Hence, a bounded estimator such as the Huber's  $t$  function may be used. The Huber's  $t$  function has a

monotonic  $\psi(e)$  function with an upper bound. The upper bound does not weight the outliers as much as the LSM technique. Other methods are serious departures from the LSM and may be employed only under dire circumstances; hence, the Huber's  $t$  function with a tuning constant of  $t = 2$  is widely used in robust estimation.

#### *Principal components estimation (PCE)*

Principal components estimation is another method of obtaining biased estimates when multicollinearity problems are prevalent in the incidence matrix. Multicollinearity may cause the process matrix to become near singular, which may result in poor estimates. Near singularity implies near zero eigenvalues and consequently a large variance and related imprecision of the least squares estimates [67]. Principal components refer to a new set of orthogonal (non-multicollinear) regressors obtained by transformation of (3.1) into a canonical form. A reduced set of the original principal components arranged in a descending order may be useful in omitting the near zero eigenvalues inducing columns of  $H$ . Appendix B describes the mathematics of the principal components estimation in detail. The bias in this form of estimation arises in setting a threshold for omitting the principal components associated with the small eigenvalues and applying least squares to the remaining regressors.

The advanced estimation methods that overcome the effects of multicollinearity in  $H$  may be employed when the statistical test returns a *NOT HIGH* level of confidence based on the structure of  $H$ . In a case of a *HIGH* level of confidence, these tests may be used as a method of model verification; running the above tests with no bias on the original data set should return the same estimates as that of least squares in order to conclude validity of the model.

An illustrative numerical example explaining the above methods of estimation may be useful for the understanding of the reader.

#### Example 3.1

Consider a system of four equations in three unknowns described by the following set of equations:

$$\begin{aligned} a + b + c &= 2 \\ a + b &= 3 \\ b + c &= 0 \\ 2a + 2b &= 6 \end{aligned} \tag{3.9}$$

The corresponding process matrix  $H$ , the observation vector  $z$ , and the vector of estimates  $x$  for the above overdetermined system of equations are

$$H = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 2 & 2 & 0 \end{pmatrix}, \quad x = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \quad z = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 6 \end{pmatrix}. \tag{3.10}$$

The process matrix  $H$  is tested for statistical confidence of estimation by performing the test described in Section 3.3. The results of the test are presented in Table 3.3.

TABLE 3.3

Boolean flags for statistical confidence of estimation in Example 3.1

| Statistical quantity | Numerical value | Boolean flag |
|----------------------|-----------------|--------------|
|----------------------|-----------------|--------------|

|                   |   |   |
|-------------------|---|---|
| VIFs              | [4, 3, 2]   | 0 |
| Rank ( $H$ )      | 3   | 0 |
| Eigen analysis    | Cond ( $H$ ) = 15.1283                                  | 0 |
| Willan-Watts test | $\Delta(\text{Confidence Interval length}) = 59.1752\%$ | 1 |

According to the results shown in Table 3.3, the level of the statistical confidence of the estimation is *NOT HIGH*. However, it is expected that since the change in confidence interval length occurs close to the Willan-Watts threshold of 50%, the LSM technique may yield estimates that are not very poor. The estimates obtained using the LSM technique are

$$\hat{x} = (H^T H)^{-1} H^T z = \begin{pmatrix} 6 & 6 & 1 \\ 6 & 7 & 2 \\ 1 & 2 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 17 \\ 17 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}. \quad (3.11)$$

The results of the LSM technique can be compared with the results of advanced methods of estimation such as robust estimation, principal components estimation, and ridge estimation. Since the change in confidence interval length by Willan-Watts test is not profound, a function with minimum departure from the least squares function is preferred in robust estimation. The Huber's  $t$  function, which does not weight the outliers as much as a least squares function, is chosen for robust estimation with a tuning constant of 2. The results of the robust estimation are

$$\hat{x}_{\text{robust}} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}. \quad (3.12)$$

In the Example 3.1, the weights in robust estimation are not altered since the problem at hand is not ill-conditioned. Hence, the estimates remain the same as those obtained using the LSM technique.

Principal components estimation may be performed on Example 3.1 as model verification procedure. The eigenvalues of the process matrix  $H$  in correlation form are,

$$\lambda = \begin{pmatrix} 0.159 \\ 0.4359 \\ 2.4051 \end{pmatrix}. \quad (3.13)$$

Condition indices, which are the ratio of the individual eigenvalues to the minimum eigenvalue, may be used to determine a threshold bias in choosing the principal components. A general rule of thumb is to set the bias at 50% of the condition number threshold used in the test for statistical confidence. Hence, principal components that possess condition indices of numerical value greater than 50 may be discarded. The remaining principal components can be used in estimation. Following the above rules, the estimates of  $a$ ,  $b$ , and  $c$  in the Example 3.1 are

$$\hat{x}_{\text{pce}} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}. \quad (3.14)$$

The above result does not differ from the results obtained using the LSM because all the condition indices fall within the limiting bias of 50. The similarity in estimated parameters obtained using the LSM and the PCE techniques can be considered as a validation of the descriptive model for the problem in Example 3.1.

Both the robust estimation using Huber's  $t$  function and the PCE technique present a mild deviation from least squares with a small bias. However, ridge estimation introduces a heavy bias in estimation and care must be employed in choosing the biasing parameter  $k$ . In order to choose  $k$  a ridge trace of the regressors is required to be plotted. Fig. 3.2 shows the ridge trace of the regressors in Example 3.1.

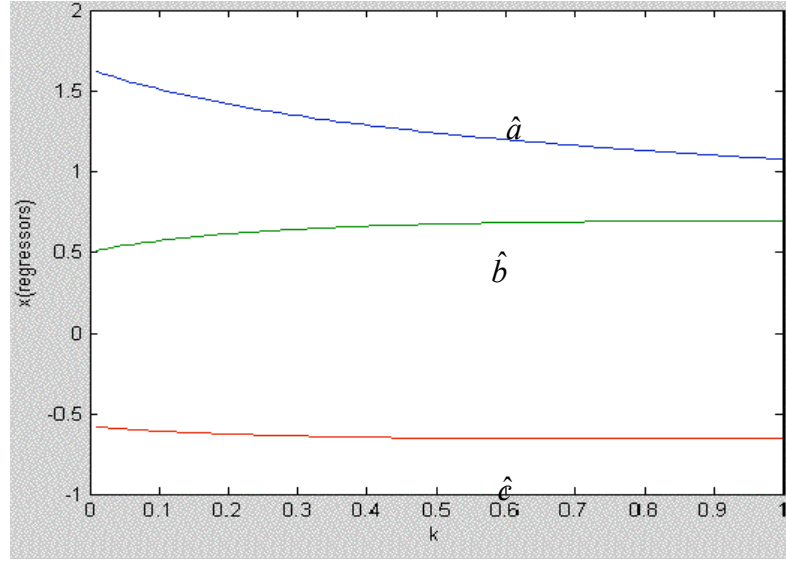


Fig. 3.2 Ridge trace of regressors versus biasing parameter in Example 3.1

The ridge trace does not reveal marked instability among the magnitude and signs of the regressors. Since, there is no apparent significant change in the magnitude of the regressors beyond a  $k$  value of 0.25, the analyst may use a biasing parameter of 0.25 for determining the biased estimates. The ridge estimates using  $k = 0.25$  for the Example 3.1 are,

$$\hat{x}_{ridge} = \begin{pmatrix} 1.3764 \\ 0.6248 \\ -0.6339 \end{pmatrix}. \quad (3.15)$$

There is significant difference in the estimates obtained using ridge estimation and LSM technique. This difference is attributed to the high value of bias in estimation. The user is advised to employ the ridge estimation only if there is an apparent change in magnitude and sign of the regressors in the ridge trace or if more than one flag is set to unity in the statistical test for confidence in estimation. A comparison of the variance among the estimates in estimation is provided in Table 3.4.

TABLE 3.4  
Comparison of variance of estimates among different methods of estimation for Example 3.1

| Method of estimation | Variance among estimates |
|----------------------|--------------------------|
| LSM technique        | 2.333                    |
| Robust estimation    | 2.333                    |
| PCE technique        | 2.333                    |
| Ridge estimation     | 1.0318                   |



Even though the variance among estimates is slightly reduced in ridge estimation, the high bias prevents the use of ridge estimation in this particular example. LSM technique may be used to obtain fair estimates of the parameters while the results of robust estimation and PCE technique can be used in establishing validity to the proposed model of system equations.

### 3.5. Considerations in loop flow model verification

A design model is always subjected to model adequacy tests and validation prior to use in practical situations. The model adequacy tests are performed to determine if the model fits the data to some accuracy. Methods such as analysis of residual plots and normal probability plots are effective tools to conclude adequacy of a state estimator. Model validation is a process that differs from model adequacy tests on a philosophical basis: validation is done by checking the model performance in diverse data ranges. The justification for conducting such validations is that the designer has little control over the behavior of the model in practical circumstances [67]. Traditional methods of model validation include techniques such as data splitting, checking with new data, and analysis of the regression coefficients based on the physical knowledge of the system [67].

In the loop flow case, the above techniques of model validation are not useful. Since each measurement of the branch difference flows is a unique set, it is not advised to split the data or seek fresh values for model verification. Each set of data corresponding to a schedule in the system may be used as new data for checking validity. Hence, a balance is sought between model adequacy tests and model validation. The designed model is subjected to model adequacy tests and judgment on validity of the model is made. Simulated test data may be used for checking the adequacy of the model. For this purpose, a bandlimited white Gaussian noise of known Signal to Noise Ratio (SNR) may be added to the branch difference flow measurements. Estimation and model adequacy tests can be performed on this new data set. The resulting residual plots and normal probability plots may be compared with those plots obtained with the original data. Any discernibly large variation in the shape of the plots may indicate an inadequacy or invalidity of the model [9]. Another method of establishing validity to a model is by carrying out different methods of estimation and comparing the deviation in the estimates. A small deviation in variance of the estimates indicated a stable model and can be used as a test of model verification.

Following the estimation of the minor loop flows from a descriptive model, a suitable method for accommodation of the estimated minor loop flows among the participating utilities is required to be designed. An algorithmic procedure for determining a charge or compensation for participating utilities based on a *take-or-pay* method is developed in the following chapters.

## CHAPTER 4

### ACCOMMODATION OF UNSCHEDULED FLOWS IN A GENCO PERSPECTIVE

#### 4.1 Need for accommodation of unscheduled flows

Loop flows in the system are caused by active power flowing on transmission circuits according to physical laws rather than according to prearranged schedules. The resultant USF can potentially cause adverse effects to the operation of the system and the market by reducing the ATC, overloading lines due to congestion, potentially degrading security and reliability, causing deviation from set market prices, and possibly resulting in forced participation of third parties in a transaction. The loop flows can be controlled by the installation of control devices such as phase shifting transformers, FACTS devices, and flowgates that restrict the amount of power flowing in a transmission circuit. However, the employment of such dedicated devices may be cost prohibitive and the installation of these controls at selected parts of a wide area system by certain players alone may introduce market pricing problems related to monopoly. The electric power industry practices control of loop flows only when the USF causes concerns of introducing congestion in the system. The control of loop flow is also done only on historically classified paths, 'qualified paths,' known to be prone to USF. In the WECC network, a qualified path is described as a path with a historical record of at least 100 hours during the past 36 months with path loading of more than 97% of path transfer capability causing curtailment of schedules due to USF in a particular direction only [4]. This procedure is selective and addresses the USF problem from a TRANSCO perspective. An alternative to the control of loop flows is the accommodation of loop flows from a GENCO perspective. Under the philosophy of accommodation, it is desired to design a method of accommodation of the loop flows based on the participation of utilities in a given USF scenario. One such method of accommodation is by the design of a contribution matrix using state estimation techniques and tagging the schedules. Individual schedules of the utilities are tagged in order to estimate the participation of each utility in a loop flow situation. In effect, the philosophy is to assign a part of the transmission charges to the utilities that load the transmission paths. The following sections describe an algorithmic approach to design a contribution factor and charge or compensate utilities based on the individual participation.

#### 4.2 A contribution factor for participating GENCOs

The loop flow problem is conceptualized as a linear estimation problem as described in Chapter 3. The minor loop flows can be estimated using different state estimation techniques depending upon the problem at hand. The estimates of the minor loop flows can be used in the design of a contribution factor for establishing participation of utilities toward USF. The contribution factor is to be appropriately weighted to obtain the participation of the utilities. The loop flows that are estimated need to be priced according to some method so that the participating utilities that are responsible will be charged proportionately. Also, any third party which is forced to participate in the loop flow scenario needs to be compensated fairly. To achieve this goal, a mathematical formula which calculates the charge or compensation for the utilities based on their participation in loop flow circumstances is developed. This formula, called the weighted contribution factor, can be used as market tool to accommodate loop flows. Prior to the development of the formula, it is desired to provide some selected definitions related to transactions and schedules, as shown in Table 4.1 [6]-[8].

TABLE 4.1  
Definition of terms related to transactions and schedules

|                               |  |
|-------------------------------|--|
| <i>Transaction / Schedule</i> | Scheduled transmission of a specific quantity of power from a supplier (generating utility) to a demand center (load point) through a particular path in the interconnected system, specified by a central management agency such as an ISO. |
| <i>Schedule tag</i>           | Electronic form which identifies agreed to schedules, the corresponding source, sink, and paths.   |
| <i>Transaction set</i>        | A transaction set refers to all the transactions occurring between an individual supply center and all the concerned load centers. The number of transaction sets in a network equals the number of players (GENCOs)                         |
| <i>Aggregate schedule</i>     | Resultant of all transaction sets occurring between all supply centers and all demand centers. Also, it represents all the flows in the system when all the players (utilities and load customers) are engaged in the market scenario.       |

The formula for the individual contribution of a utility toward loop flows in a network is developed by tagging the schedules in the system. The schedule tags of each transaction set and the aggregate schedule are used to obtain the unscheduled flow occurring in the transmission circuits. The USF corresponding to the aggregate schedule is obtained as a difference between the measured (or observed) actual flow in the circuits and the desired aggregate schedule. The actual flow occurring during the individual transaction sets cannot be observed (or measured) in a system: hence, they are estimated using power flow software with the corresponding schedule tag as an input. The USF assumed to occur in the circuits during the individual transaction sets are then obtained as the difference between the estimated actual flow and the schedule tag. The number of schedule tags for the transaction set equal the number of GENCOs participating in the USF scenario. The estimates of minor loop flows occurring during the aggregate schedule and during each of the transaction sets can be obtained using an appropriate technique of estimation. The estimated values of minor loop flows during the aggregate schedule are assumed to be a  $nx1$  column vector named  $x_{all}$  where  $n$  is the number of minor loop flows in the system. The  $nx1$  column vectors of minor loop flows estimated during each of the transaction sets are named  $x_1, x_2, \dots, x_m$  where  $m$  represents the corresponding player (GENCO) index in the system participating in the loop flow scenario. The contribution factor matrix, of order  $m \times n$ , is obtained by the array division of each minor loop flow estimate during the corresponding transmission set tag  $x_1, x_2, \dots, x_m$ , by the minor loop flow estimate during the aggregate schedule tag,  $x_{all}$  [6]-[8]. The weights are added to each of the rows by multiplying the ratio of the generation of the utility  $m$ ,  $Gen\ m$ , to the total generation in the system, represented as  $sum(Gen)$ . The contribution factor of the  $m^{th}$  utility toward the  $n^{th}$  estimated minor loop flow,  $CF_{mn}$ , is given by the element in the  $m^{th}$  row and  $n^{th}$  column of the weighted contribution factor matrix,

$$CF_{mn} = \frac{x_{all\ n} - \left( \sum_{\substack{i=1 \dots m \\ i \neq m}} x_{i\ n} \right)}{x_{all\ n}} \times \frac{Gen\ m}{sum(Gen)}, \quad (4.1)$$

where  $x_{all\ n}$  refers to the  $n^{th}$  estimated minor loop flow in the aggregate schedule and  $x_{i\ n}$  refers to the  $n^{th}$  estimated minor loop flow in the  $i^{th}$  transaction set. The individual contribution of a util-

ity toward any minor loop flow can be obtained from this weighted contribution factor matrix by summing the individual contribution factors of each utility toward each estimated minor loop flow. The total contribution of a utility  $m$ ,  $CF_m$ , toward all the  $n$  estimated minor loop flows in a system is given by the column sum of the contribution factor matrix,

$$CF_m = \sum_{i=1}^n col_i(CF_{m \times n}) = \sum_n \left( \frac{x_{all\ n} - \left( \sum_{\substack{i=1 \dots m \\ i \neq m}} x_{i\ n} \right)}{x_{all\ n}} \right) \times \frac{Gen\ m}{sum(Gen)}. \quad (4.2)$$

The numerator of the contribution factor formula has been adjusted to accommodate the inapplicability of the superposition principle for power flows in circuits. The minor loop flows in the system due to a particular player,  $m$ , is obtained as the difference between the minor loop flows occurring during the aggregate schedule and sum of the minor loop flows occurring during transaction sets of all the players except  $m$ .

The array division of minor loop flows in determining contribution factors might make the numerical process cumbersome. Also, (4.2) may fail to yield a finite contribution factor if even one of the estimates of the minor loop flow in the aggregate schedule is near zero. In practical cases, estimates of the minor loop flows in an aggregate schedule may be near zero during instances such as the flow on similar parallel lines. A method to overcome the difficulties of numerical burden and nonfinite contribution factors is the Hölder norm modification of (4.2). Depending upon the type of Hölder norm modification used in (4.2), the formula can be used to determine the participation of the utility pertaining to the magnitude and direction of the minor loop flows. Table 4.2 explains the choice of Hölder norm modification and the corresponding accommodation technique.

TABLE 4.2  
Contribution formula type and corresponding accommodation

| Type of accommodation   | Hölder norm modification of (4.1)  |
|---|--|
| Direction and magnitude of all minor loop flows caused by participant $m$ | $CF_m = \frac{\left\  x_{all} - \left( \sum_{\substack{i=1 \dots m \\ i \neq m}} x_i \right) \right\ _1}{\ x_{all}\ _1} * \frac{Gen\ m}{sum(Gen)} \quad (4.3)$           |
| Magnitude of all minor loop flows caused by participant $m$               | $CF_m = \frac{\left\  x_{all} - \left( \sum_{\substack{i=1 \dots m \\ i \neq m}} x_i \right) \right\ _2}{\ x_{all}\ _2} * \frac{Gen\ m}{sum(Gen)} \quad (4.4)$           |
| Largest minor loop flow caused by participant $m$                         | $CF_m = \frac{\left\  x_{all} - \left( \sum_{\substack{i=1 \dots m \\ i \neq m}} x_i \right) \right\ _\infty}{\ x_{all}\ _\infty} * \frac{Gen\ m}{sum(Gen)} \quad (4.5)$ |

After determining the contribution factor for each utility proportionate to its participation, it is desired to establish a method of assigning a charge or compensation to the utility depending upon the contribution factor. Prior to describing an algorithm for determining a monetary value for GENCOs participating in the USF scenario in the system, an introduction to transmission pricing in deregulated electric power markets is in order.

#### 4.3 Transmission pricing in deregulated electric power markets

In a deregulated setting, power is generated by utilities (suppliers, GENCOs) and transmitted by TRANSCOs. TRANSCOs are a separate entity, contrary to a regulated environment in which any single entity may be responsible for both production and transmission. The presence of TRANSCOs increases the number of players in a deregulated market structure and hence pricing may be an important issue to reckon with market strategies. There are several methodologies that are being practiced in contemporary market to price power transmission. The most widely used methodologies are classified as rolled-in and incremental transmission pricing paradigms. The rolled-in model is often called the *allocation* model because it allocates a price for all the existing transmission system costs as well as new costs without considering their causes. The incremental pricing methodology prices only the new costs in transmission that are introduced by the customers. The existing costs remain the responsibility of the present customers. Fig. 4.1 depicts the different types of rolled-in and incremental methodologies in contemporary use. Various types of transmission pricing paradigms are described in Appendix C.

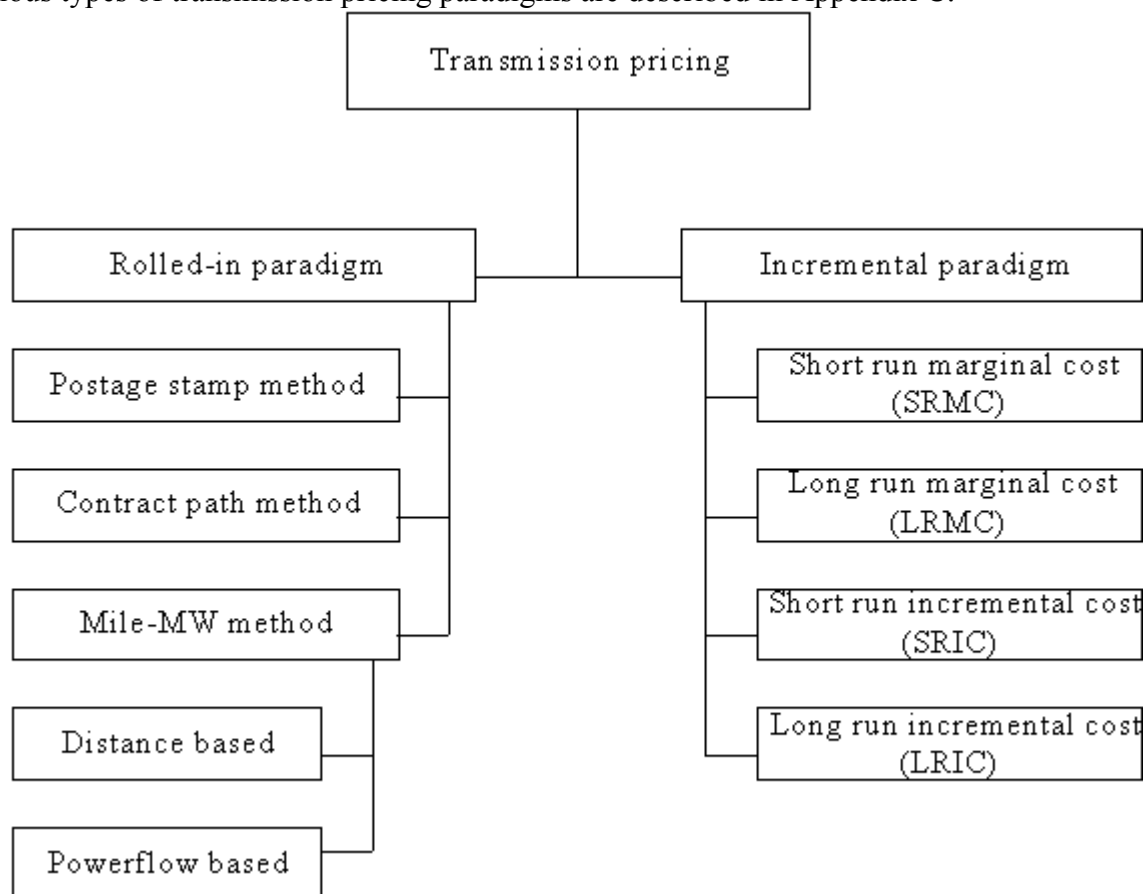


Fig. 4.1 Classification of transmission pricing methodologies

#### 4.4 Accommodation of USF using transmission pricing paradigms

The USF can be accommodated among the participating utilities (GENCOs) by employing the calculated weighted contribution factors of each utility toward the minor loop flows and any of the allocation type transmission pricing paradigms. This method establishes a monetary value to the USF based on the estimated minor loop flows in the system during the individual transaction sets and the aggregate schedule. The procedure determines a charge or compensation to the participating utilities. The unscheduled or branch difference flow is obtained as a difference between the measured actual flow and the desired flow during the aggregate schedule in the system. The deviation in cleared market price due to the USF in aggregate schedule is obtained by applying the same transmission pricing methodology as applied to the desired schedule in the cleared market. The difference cost due to the USF is the difference in the total cost associated with the actual flow and the total market cleared cost of the aggregate schedule. Depending upon the type of accommodation required by an ISO, the appropriate formula for contribution factors is chosen from Table 4.2. The monetary value due of each utility is then calculated by proportionating the difference cost due to USF in the ratio of the contribution factors of the individual utilities as,

$$USF\$_m = (CF_m) \left| \left[ \$_1 \ \$_2 \ \dots \ \$_B \right] [z_{agg}] \right|, \quad (4.10)$$

where  $USF\$_m$  refers to the monetary value associated with the  $m^{\text{th}}$  utility for participating in the USF scenario,  $\$_i$  is the price of transmission on branch  $i$  of the network, and  $z_{agg}$  is a  $(B \times 1)$  vector of USF in the system during the aggregate schedule. If (4.3) is used to accommodate the USF in the system, the  $USF\$_m$  may take either a positive or a negative value depending upon the direction of the estimated minor loop flows. A negative value of  $CF_m$  from (4.3) indicates that the estimated minor loop flows in the transaction set  $m$  are in the opposing direction than the estimated minor loop flows in the aggregate schedule. This is indicative of a reduction in the minor loop flows due to utility  $m$ ; hence, the utility  $m$  is compensated with the amount indicated by  $USF\$_m$ . However, if  $CF_m$  is a positive quantity, then the utility  $m$  is charged the amount indicated for introducing minor loop flows in the system that cause deviation from the prearranged aggregate schedule.

An algorithm for obtaining a monetary value of charge or compensation to the utility for participation in a USF scenario is described below. The algorithm, named USF accommodation algorithm, integrates the process of estimation of minor loop flows and the accommodation of USF using the estimated minor loop flows.

#### 4.5 USF accommodation algorithm

A stepwise description of the USF accommodation algorithm is given in Table 4.4. The USF accommodation algorithm is transparent to the system under consideration and can be used in a post-operative markets perspective to accommodate USF occurring in the system. The USF is accommodated by estimating values of fictitious flows in the system termed minor loop flows, which follow the path of currents in the meshes of the system. The USF accommodation algorithm is intended to be performed several times between every change of system schedules and the final monetary value due of each utility is expected to be the average of the monetary values obtained in each run. This would ensure that any system contingency occurring between sched-

ule changes is taken into account while calculating the  $USF\$$  values. Fig. 4.2 provides a flow-chart of the various steps in the USF accommodation algorithm.

TABLE 4.3  
Steps in the USF accommodation algorithm

| Step Number | Description of task  |
|-------------|--|
| 1           | The paths of the minor loop flows in a system are identified using network topology and the process matrix $H$ is formed.  |
| 2           | The electronic tags of the individual transaction sets and the aggregate schedule are procured.  |
| 3           | Measurements of the actual flow occurring during the aggregate schedule are obtained from the system.  |
| 4           | Estimates of the actual power flowing in the system during individual transaction sets are determined by running power flow software with the tags of the transaction sets as inputs.        |
| 5           | The $z$ vector of USF for the individual transaction sets and the aggregate schedule are obtained.   |
| 6           | Statistical test for confidence in estimation is performed on the process matrix $H$ and an appropriate technique to estimate minor loop flows during individual transaction sets is chosen. |
| 7           | A Kalman filter is employed to obtain estimates of the minor loop flows during the aggregate schedule. This is done to avoid the effect of noise in measurements from the system.            |
| 8           | Model adequacy is verified using normal plots, residual analysis, and by advanced estimation techniques.   |
| 9           | Minor loop flow estimates from individual transaction sets and the aggregate schedule are used to calculate the weighted contribution factor for each utility toward USF.                    |
| 10          | Choice of allocation type transmission paradigm and the accommodation technique based on the contribution factor formula are made.   |
| 11          | A monetary value corresponding to a charge or compensation due of each participating utility is determined.  |

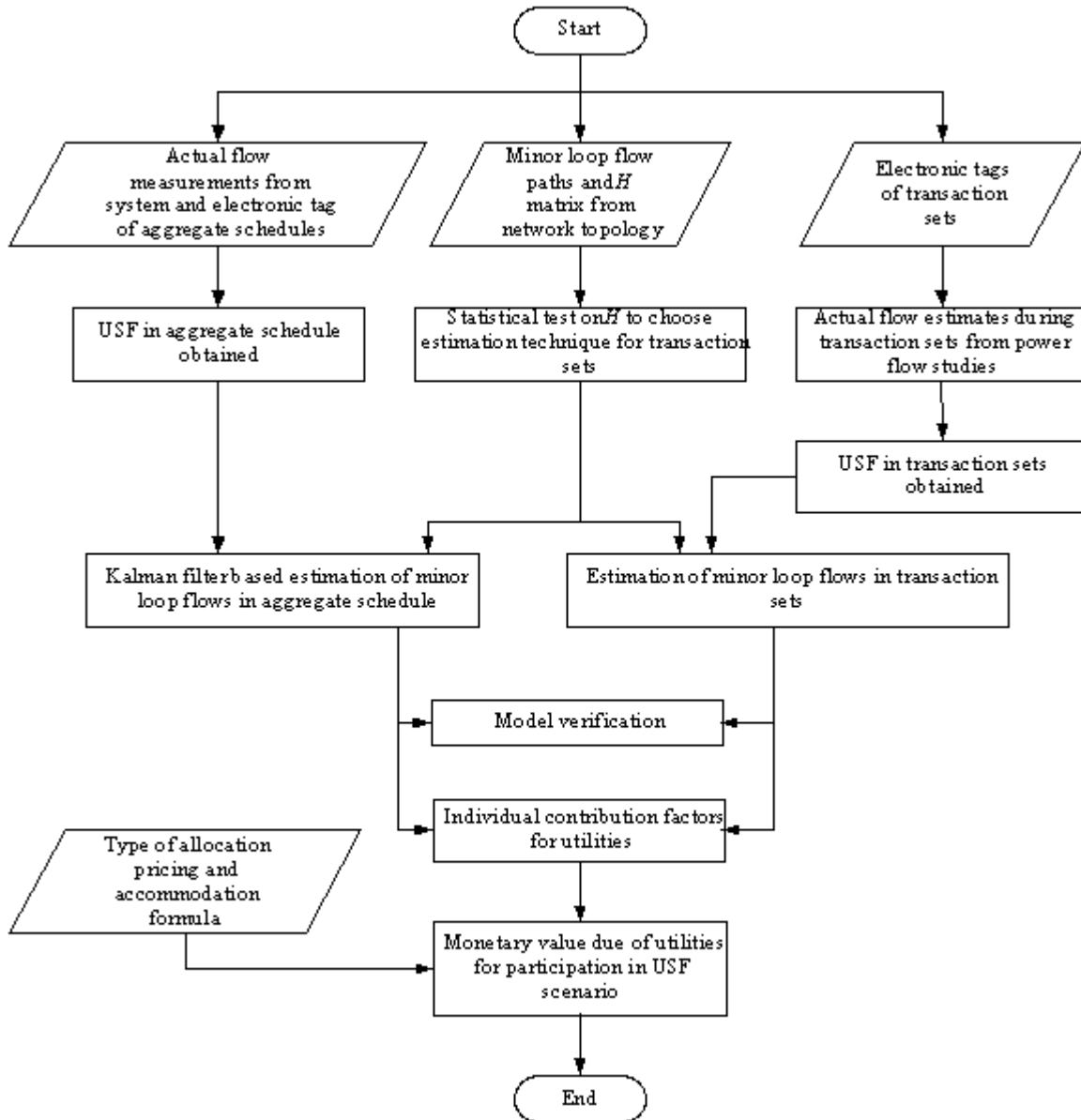


Fig. 4.2 Flowchart of tasks in the USF accommodation algorithm

The USF accommodation algorithm is a method to apportion the deviation costs occurring in the system due to USF among participating utilities by using estimates of fictitious minor loop flows. The method of apportioning a part of the transmission costs to generating utilities loading the transmission lines has the following advantages over the existing methods of accommodation of USF:

- The method is not selective and addresses all the participants proportionate to the participation.
- The method, unlike the circle diagram, is not empirical and can be applied to systems that are highly meshed
- The method accommodates pre-congestion levels of USF in the system among GENCOs.
- The contribution factor formula does not depend upon assumptions of superposition.



- The method determines an equitable monetary value for unscheduled flows in the system among participating utilities; utilities that load the transmission path are charged and for the utilities that do not participate in the USF scenario are compensated.
- The contribution factor formula is compatible with any allocation type transmission paradigm.
- The method is intended to serve as a motivation for central agencies to adopt better scheduling algorithms that abide with system requirements.

Another method of accommodating USF among utilities without considering minor loop flows or their paths is by employing game theory techniques. The following sections describe a game theory based USF accommodation technique.

#### 4.6 Game theory

Game theory is the branch of mathematics that deals with the framework for decision-making in multi-player competition. The application of game theory started with the seminal work of von Neumann and Morgenstern [70], in the 1940s and has hence been developed and applied in biological sciences, political science, and strategic defense planning among other fields. However, the single biggest application of game theory is in the field of economics. Game theory gives a mathematical representation to most of the strategies and rational decisions involved in a multi-player game. A multi-player game may be a zero sum game if the winnings and losses of all players sum to zero. Otherwise, the game is called a non-zero sum game. The outcome of a game for a player is called a payoff. A significant role of game theory is in proportionate division of profits among players based on individual strategies. A concept of a value for each player in the game was introduced by Shapley [71]. The so-called Shapley values are the expected marginal contribution of each player in a game based on the order of the coalitions formed in the game. A straightforward method of division of profits is described by the egalitarian value in which the value of a player is obtained irrespective of the coalitions formed in a game [13]. The following subsections describe the division of profits among the players using the Shapley value and the egalitarian method.

##### A. The Shapley value and the egalitarian value

The Shapley value of a player  $i$  in an  $n$ -person game determines the expected marginal contribution of the player in a game given all the coalitions possible among the  $n$  players. The Shapley values are calculated based on the game theory axioms of symmetry, efficiency, and additivity of games [71]. Each  $n$ -person game has a characteristic function,  $v(n)$  associated with it. The characteristic function,  $v(n)$  of a  $n$ -person game is defined as the mapping  $v$  to  $2^n$ . The marginal contribution of a player  $i$ ,  $\phi_i(v)$ , given all the coalitions in the game, is found using Shapley values. The Shapley values are estimated with the assumption that each player receives the extra amount brought to a coalition of players. The extra amount player  $i$  brings to a coalition  $S$  is expressed as  $v(S) - v(S \setminus \{i\})$ , where  $v(S)$  represents the total worth of the coalition  $S$  and  $v(S \setminus \{i\})$  represents the worth of the coalition in the absence of player  $i$ . The order of formation of the coalition among the  $S$  players also plays a role in the determination of the Shapley values. The player  $i$  can be preceded by  $(S-1)$  other players in  $(S-1)!$  ways while the remaining  $(n-S)$  players can succeed the player  $i$  in  $(n-S)!$  ways among the total  $n!$  possible permutations among the players. Hence, the probability that  $(S-1)$  players precisely precede the player  $i$  among  $n$  players is given by  $(S-1)!(n-S)!/n!$  [71]. The Shapley value for a player  $i$  in a coalition of players  $S$  in an  $n$ -person game is,

$$\phi_i(v) = \sum_{\substack{S \subseteq N \\ i \in S}} \frac{(S-1)!(n-S)!}{n!} [v(S) - v(S \setminus \{i\})], \quad (4.11)$$

where  $N$  represents the universal set of  $n$  players.

Consider a three player game with  $N = \{A, B, C\}$  and  $S = \{1, 2, 3\}$  or  $S = \{A, B, C, AB, AC, BC, ABC\}$ . The characteristic function of the game is given by  $v(n)$  and the core of the game corresponding to the coalitions is  $C = \{v(A), v(B), v(C), v(AB), v(AC), v(BC), v(ABC)\}$ . The Shapley value of the players  $A, B$ , and  $C$  in the game can be expressed in the matrix form as,

$$\begin{bmatrix} \phi_A \\ \phi_B \\ \phi_C \end{bmatrix} = \frac{1}{3!} \begin{bmatrix} 0!2! & 1!1! & 2!0! \end{bmatrix} \begin{bmatrix} v(A) & v(AB)+v(AC)-v(B)-v(C) & v(ABC)-v(BC) \\ v(B) & v(AB)+v(BC)-v(A)-v(C) & v(ABC)-v(AC) \\ v(C) & v(AC)+v(BC)-v(A)-v(B) & v(ABC)-v(AB) \end{bmatrix}^T, \quad (4.12)$$

where  $\phi_A$ ,  $\phi_B$ , and  $\phi_C$  represent the share of players A, B, and C in the 3-person game computed using the Shapley value method. The egalitarian value of a game does not count the coalitions; instead, the total worth of the game is distributed in proportion to the participation of the players [13].

Game theory has been applied to the deregulated electricity markets to analyze transactions, pricing electricity in power pools, market modeling, designing GENCO and bidding strategies, spot market pricing, transmission expansion planning, congestion management, and modeling price dynamics [84]-[92]. A new application of game theory to accommodate USF among GENCOs in a deregulated electric market is proposed and compared with the contribution factor method.

#### 4.7 Game theory based accommodation of USF among GENCOs

Game theory methods can be employed to apportion the deviation of costs among utilities in a USF scenario. The Shapley and egalitarian values can be used to predict the expected marginal contribution of a utility toward the USF in a system. This method of accommodation of USF among utilities is transparent to the incremental matrix, the transmission operators, the type of USF, and the paths of the USF. The method however is dependent on tagged schedules. The axioms of symmetry, efficiency, and additivity of the game are satisfied in a USF scenario.

In a wide-area system subjected to USF, the schedule tags and the aggregate schedule are procured from a central scheduling agency like an Independent System Operator (ISO). The number of transaction sets equals the number of players (utilities) in the power market. The measurements of the actual power flows in the transmission circuits of the system during the aggregate schedule are acquired. The actual flows occurring in the system circuits during the individual transaction sets are obtained as in the contribution factor method; by performing load flows using the corresponding tagged schedules as inputs. The difference between the actual power flows and the desired scheduled flows can be attributed to the USF occurring in the system. The deviation in cost from the market-cleared amount is calculated using any allocation type transmission pricing paradigm like the Mile-MW method or the postage stamp method [6]. This difference in cost incurred between the actual power flows and the desired power flows forms the characteristic function of the game,  $v(n)$ . The individual characteristic functions of the players and the combined characteristic function for the coalition of players are obtained from the difference costs during the transaction sets and the aggregate schedule respectively. The individual characteristic functions form the base winnings of the player in any coalition. The Shapley values are computed and added to the base winnings to determine the expected marginal

contribution of each player to the USF scenario. The marginal contributions of the players are expected to lie within the core of the game. The egalitarian value is determined by proportionating the total worth of the game with respect to the generation of each player in the system. The total worth of the game is described by the characteristic function of the coalition that involves all the players in the system. The values of the players obtained using each of the methods need not necessarily be similar. In such cases of variation, it is advised to choose the most appropriate method depending upon the importance of the arrival of a player in any coalition [13].

The game theory based methods of accommodating the USF among GENCOs work entirely on a markets perspective and do not take into consideration the fictitious loop flows in a network. The cost deviations in a wide-area system occurring due to the presence of USF may be accommodated among the participating utilities by employing any of the above described methods. However, it is prudent at this juncture to provide a relationship between the different cost apportioning methods. In a  $m$  utility USF game, let  $\phi_{m \times 1}$  represent the Shapley values based cost vector for the participants,  $v(m)$  represent the characteristic function of the game (or the deviation in cost due to USF when all  $n$  players are involved in the USF circumstance), and  $CF$  be the  $m \times 1$  vector of the ratio of weighted contribution factors for the  $m$  participants. The Shapley values, egalitarian values, and the contribution factors are related such that,

$$\begin{bmatrix} 1 & 1 & \dots \end{bmatrix}_{\times m} \begin{bmatrix} \phi(1) \\ \phi(2) \\ \vdots \\ \phi(m) \end{bmatrix}_{m \times 1} = v(m) = \begin{bmatrix} 1 & 1 & \dots \end{bmatrix}_{\times m} \begin{bmatrix} CF_1 \\ CF_2 \\ \vdots \\ CF_m \end{bmatrix}_{m \times 1} v(m) \quad (4.13)$$

The term  $(CF)_{m \times 1} v(m)$  represents the vector of monetary values associated with the individual players (GENCOs) in the system and can be obtained from (4.10). The choice of the technique to accommodate USF among the GENCOs is decided by the central agency, ISO.

The development of a user friendly menu driven Graphical User Interface (GUI) that assigns charges or compensation to participating utilities using different techniques of accommodation is described in the following chapters along with case studies on several test systems.

## CHAPTER 5

### ILLUSTRATIVE CASE STUDIES OF LOOP FLOW ESTIMATION AND ACCOMMODATION IN TEST SYSTEMS

#### 5.1 Introduction

Case studies on test systems are used to illustrate the design and validation of a model describing the unscheduled flows in terms of the fictitious minor loop flows. Upon establishing validity of the model, it is used for the estimation of the minor loop flows in the system during the aggregate schedules and the corresponding transaction schedules. The estimates of the minor loop flows from the schedule tags are used in the accommodation of the unscheduled flows using appropriate techniques of accommodation. An alternative method of accommodation of the USF is performed by employing game theory techniques. The different test systems used for illustrating the accommodation of USF are the following:

- 9 bus 3 generator test system
- Modified IEEE 30 bus test system [76] and
- IEEE 57 bus test system [76].

The following sections describe the case studies for the accommodation of USF in each of the above systems.

#### 5.2 Case study on a 9 bus test system

The 9 bus test system used for the estimation and accommodation of loop flows has 12 branches with generation utilities situated at bus numbers 1, 3 and 7 and load centers located at bus numbers 2, 4, 5, 8 and 9. The line parameters for the system are shown in Table D.1 of Appendix D. Fig. 5.1 represents the test system with the generation, loads, and the desired flows in transmission circuits during the aggregate schedule. Since the system has three GENCOs, there are three electronically tagged transactions sets, each corresponding to a utility and the respective loads served.

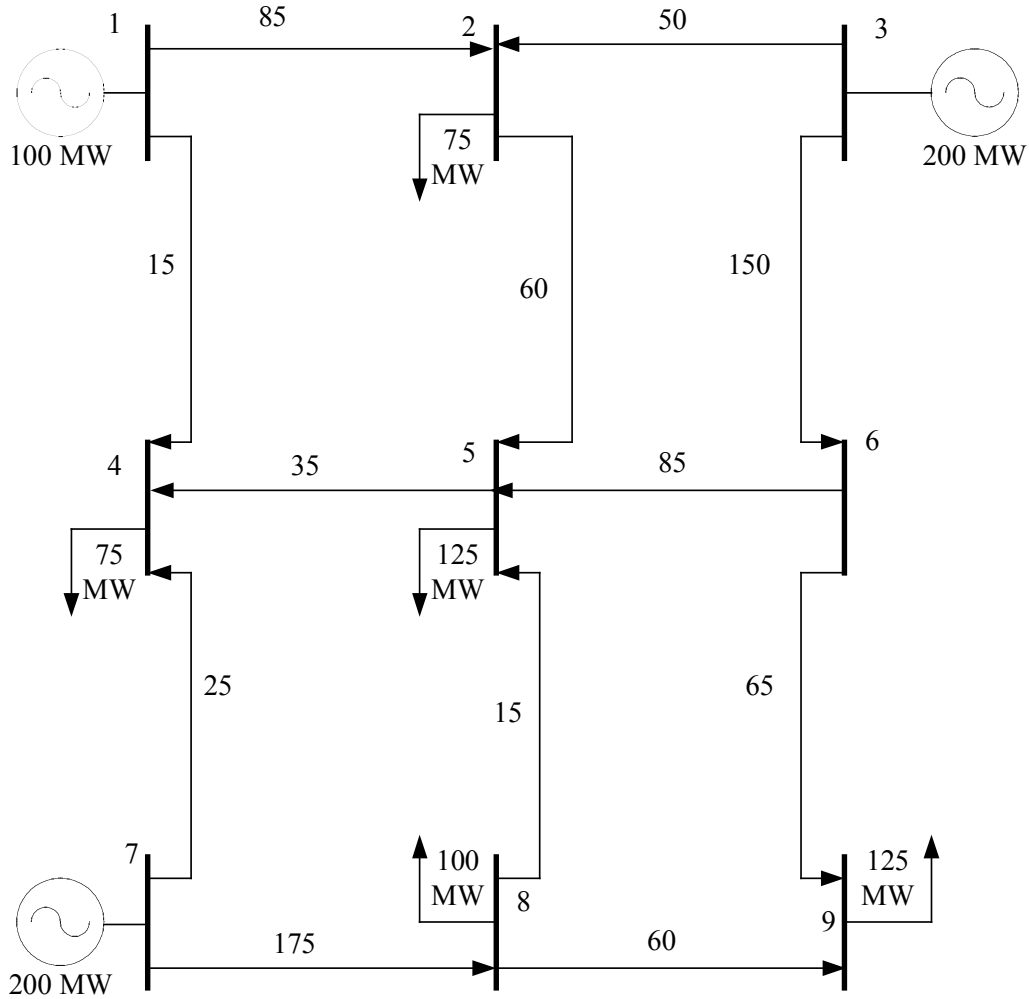


Fig 5.1 9 bus test system with generation, loads, and desired flows in aggregate schedule

An ISO is assumed to assign the aggregate schedule and the transaction sets based upon the generation schedules and the load demand in the system shown in Tables D.2 and D.3 respectively. The desired active power flow in the system during the aggregate schedule is depicted in Table D.4. Generation and corresponding load schedules for the transaction sets are as shown in Table D.5 with each schedule tag represented by the corresponding column.

Due to the fact that power flow can not be directed according to schedules, actual power flows deviate from the schedules thus giving rise to the phenomenon of USF. The USF or the branch difference flows are assumed to be a consequence of the minor loop flows in the systems which circulate in paths restricted to circuit meshes. The paths of the minor loop flows in the 9 bus test system, represented by  $L_1$ ,  $L_2$ ,  $L_3$ , and  $L_4$ , in Fig. 5.2, are determined using the network topology.

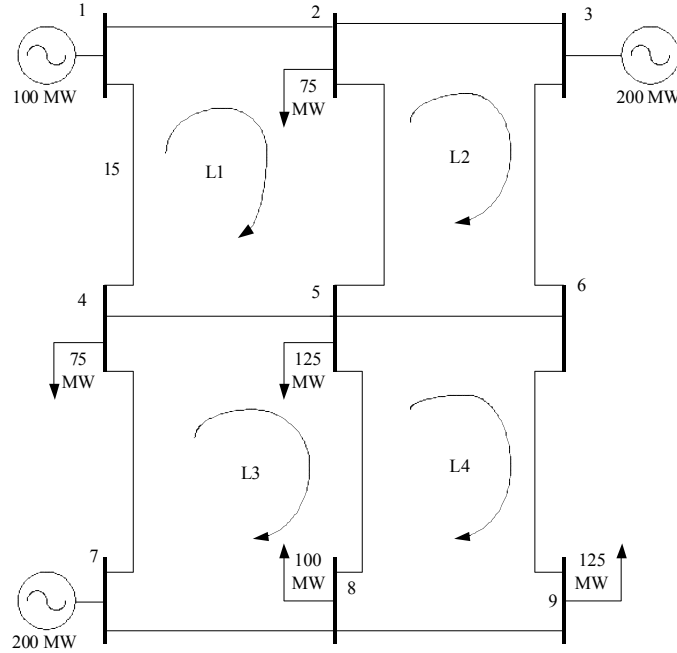


Fig 5.2 Minor loop flow paths in 9 bus test system

From the knowledge of the paths of the minor loop flows, the process matrix  $H$  that describes the branch difference flow (USF) in terms of the fictitious minor loop flows can be designed. The process matrix can be subjected to the statistical test described in section 3.3 to determine a level of confidence in estimation. Table 5.1 depicts the status of the individual test flags and the level of confidence outputted by the statistical test.

TABLE 5.1

Output of the statistical test for the process matrix of the 9 bus test systems

| Flag                | Status   |
|---------------------|----------|
| VIF flag            | 0        |
| Eigenanalysis flag  | 0        |
| Rank flag           | 0        |
| Willan-Watts flag   | 1        |
| Level of confidence | NOT HIGH |

The statistical test indicates a '*NOT HIGH*' level of confidence in the estimation process; this is because of the Willan-Watts flag being set to a value of 1 indicating a minor multicollinearity defect in the structure of the process matrix. Since at least one flag has been operated by the statistical test, the multicollinearity characteristics of the process matrix need to be analyzed. Multicollinearity in process matrix may destroy confidence in estimates when an unbiased estimation technique is used. For this purpose, advanced methods of biased estimation such as robust estimation, ridge regression, and principal components estimation may be sought as a remedy. A ridge trace of the regressors (minor loop flows) in the 9 bus test system depicting the variation of the values of the estimates with the bias in estimation is shown in Fig. 5.3.

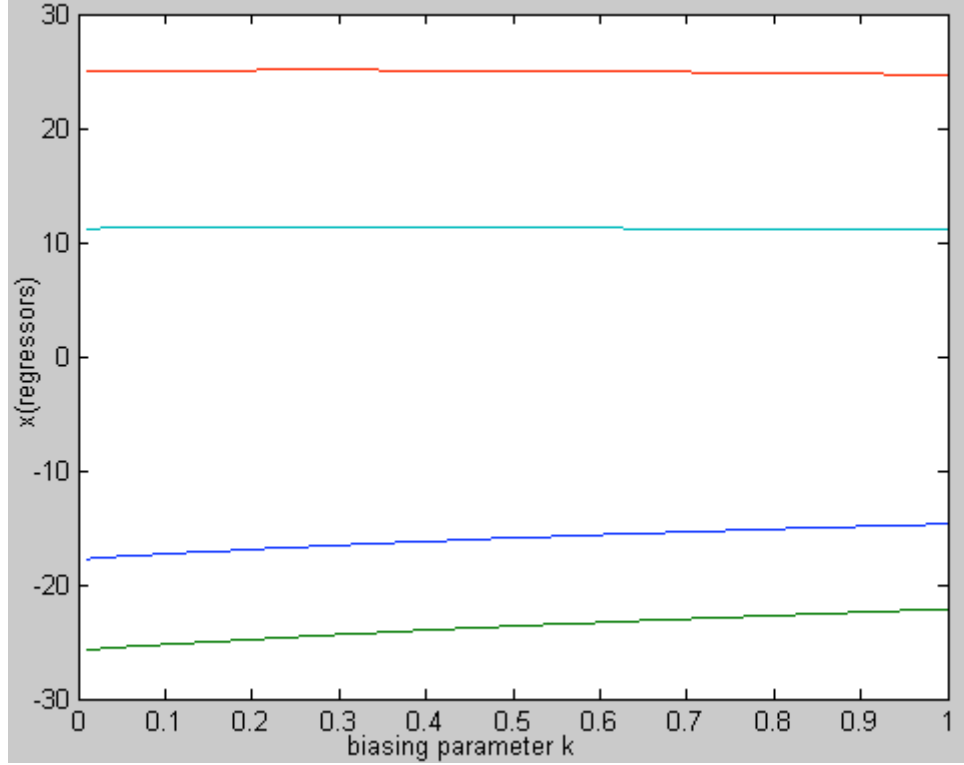


Fig. 5.3 Ridge trace of the regressors for the 9 bus test system

From the ridge trace in Fig. 5.3, it can be seen that the regressors do not change signs or show huge variation in magnitude with change in biasing parameter; this implies minor issues of multicollinearity in the process matrix  $H$ . This is also corroborated by the fact that the Willan-Watts test statistic for the process matrix is, 52.762 %; this is not dramatically different from the test statistic threshold value of 50 %.

An implication of the above results is the redundancy of ridge estimation in obtaining the minor loop flow estimates for the 9 bus test system. It is expected that the unbiased least squares estimator may yield similar estimates as the advanced estimation techniques. The redundancy may be gainfully employed as a validation technique in the absence of other validation methods such as data splitting and gathering new data.

The actual flow occurring in the system during the aggregate schedule is obtained as measurements from the system while those occurring during the individual transaction sets are obtained by performing load flow studies. The USF on the transmission circuits during aggregate schedule and the individual transaction sets are obtained by the difference between the corresponding actual flows and the scheduled flows. The USF corresponding to each tag is used as the measurement vector  $z$  in the linear model for the estimation process.

The actual power flowing during the aggregate schedule is usually a set of noisy measurements obtained from the system. However, since the case study is performed on a test system, the observables are not readily available. Hence, simulated data sets with random white noise components in the measurements of the aggregate schedule are generated by computer programs. Estimation of minor loop flows from noisy measurements of actual power flow can be performed using a discrete Kalman filter using a GM process as a model. The GM process is assumed to have unity variance and time constant with very large error covariance (of the order

of  $10^{10}$ ). The variance of the error of the measurement process is also assumed to be unity. The discrete Kalman filter algorithm is iteratively performed to obtain noise free estimates until the relative change in the norm of the estimation error is less than a preset threshold of 1%.

The estimates of the minor loop flows in the aggregate schedule obtained from different estimation techniques such as the least squares, ridge estimation, PCE, robust estimation, and discrete Kalman filtering can be used as validation for the model adequacy. Similarity in the estimates of the minor loop flows obtained from the different estimation techniques establishes validity of the model. Fig. 5.4 depicts a histogram plot of the estimates of the minor loop flows in the aggregate schedule of the 9 bus test system using different estimation techniques.

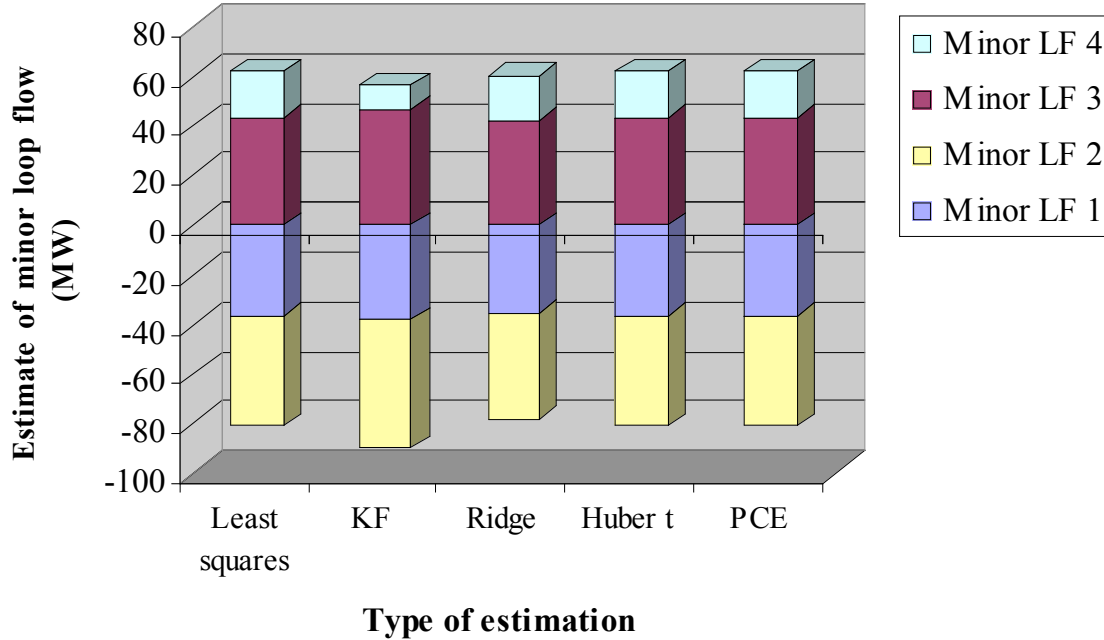


Fig. 5.4 Comparison of estimates of minor loop flows in the aggregate schedule of the 9 bus test system using different techniques of estimation

The minor loop flow estimates in the aggregate schedule with uncontaminated measurements are the same when estimated using the least squares method, the Huber  $t$  type robust regression, and the principal components estimation technique. The estimates of the minor loop flows in the aggregate schedule with random white noise contamination are obtained using the discrete Kalman filter. The relative error between estimates obtained using the discrete Kalman filter and the least square estimates of uncontaminated measurements is 0.1619. The relative error between estimates obtained using the unbiased least square method and the biased ridge estimation is 0.0309. The low relative error between the least squares and the other estimation techniques is a motivation for employing the least squares technique for the estimation of the minor loop flows in the 9 bus test system. This establishes the validity of the linear model that relates the minor loop flows and the USF in the 9 bus test system. The same model may be employed to estimate the minor loop flows in the individual transaction sets of every GENCO in the 9 bus test system. The estimate of the minor loop flows in the aggregate schedule and the transaction sets of the 9 bus test system may now be used to determine a contribution factor for each GENCO.



The contribution factor may be determined depending upon the method of accommodation sought according to Table 5.2. The take or pay charge is designed by using the contribution factors in conjunction with an allocation type transmission pricing paradigm. In the example of the 9 bus test system, the distance based mile-MW method of transmission pricing at the rate of \$3 per mile per MW is employed. The difference between the market-cleared transmission price of the desired flows and the newly incurred transmission price for the actual flows in the aggregate schedule is determined. The difference in the transmission pricing introduced by the unscheduled flows can be apportioned among the GENCOs in the ratio of the individual contribution factors. A positive value of the take or pay charge represents a charge due of the GENCO toward overloading the transmission system with USF; a negative value of the take or pay charge denotes a compensation received by the GENCO for participation in the USF scenario. Table 5.2 lists the take or pay charge due of each utility in the 9 bus test system for a given set of aggregate schedule and transaction sets. The take or pay charge of each GENCO may be considered as signal for the respective utility to decrease loop flow in the system; this can be done by changing the generation schedule to minimize the contribution factor. Also, ISOs may adapt better scheduling techniques to reduce the total difference cost associated with the USF in the system.

TABLE 5.2

Accommodation type and corresponding take or pay charge for the GENCOs in the 9 bus test system

| Utility                    | Contribution factor based take or pay charge (\$) |                               |                                    |
|----------------------------|---|-------------------------------|------------------------------------|
|                            | $\ L_1\ $ based accommodation                     | $\ L_2\ $ based accommodation | $\ L_\infty\ $ based accommodation |
| 1                          | 9253.36   | 9270.48                       | 10360.32                           |
| 3                          | 10110.28  | 9479.27                       | 7436.04                            |
| 7                          | 21731.86  | 22345.75                      | 23299.13                           |
| Total difference cost (\$) |   | 41095.50                      |                                    |

Another method of accommodation of USF which does not take into consideration the paths of the minor loop flows is the game theory based apportioning of charges. The Shapley value technique may be sought to apportion the difference cost due to USF among the participating GENCOs. In this method of cost apportioning, the difference in transmission pricing associated with each transaction set as well as the aggregate schedule is determined. The difference cost corresponding to a transaction set is considered as the worth or contribution of the respective GENCO in the USF game. The total worth of the coalition of the utilities is the difference cost associated with the aggregate schedule. The Shapley values determine the marginal contribution of each player in a game depending upon the order of formation of the coalitions. Table 5.3 depicts the marginal contribution of each GENCO toward the total difference cost in the aggregate schedule based on Shapley values.

TABLE 5.3

Shapley value based contribution of the GENCOs in the 9 bus test system

| Utility | Shapley value based charge (\$) |
|---------|---------------------------------|
| 1       | -91735.50                       |
| 3       | 101301.75                       |
| 7       | 31529.25                        |

|                            |          |
|----------------------------|----------|
| Total difference cost (\$) | 41095.50 |
|----------------------------|----------|

The Shapley value based costs for GENCOs toward accommodation of the USF is different from the contribution factors based costs. This is due to the fundamental difference in the approach toward the problem in the two methods. The contribution factor method is a system based post operative technique that assumes the presence of fictitious minor loop flows in a wide area system and then proceeds to minimize the USF through estimation techniques. The game theory based method is an entirely market based technique which divides the profits among the players in the order of the formation of coalitions among players. Also, the game theory method involves the determination of transmission pricing for the USF associated with each transaction set. For the above reasons, it is envisioned that the game theory technique of accommodation of USF may not be as suitable as the contribution factor based accommodation. Hence, for the application of accommodating USF in wide area systems among participating GENCOs, the contribution factor technique is recommended.

The following sections describe the illustrative case studies of accommodation of USF in the modified IEEE 30 bus test system and the IEEE 57 bus test system.

### 5.3 Case study on the modified IEEE 30 bus test system

The modified IEEE 30 bus test system has 41 transmission circuits and 3 utilities situated at Glen Lyn (bus 1), Claytor (bus 2), and Cloverdale (bus 28) as shown in the one-line diagram in Fig. 5.5.

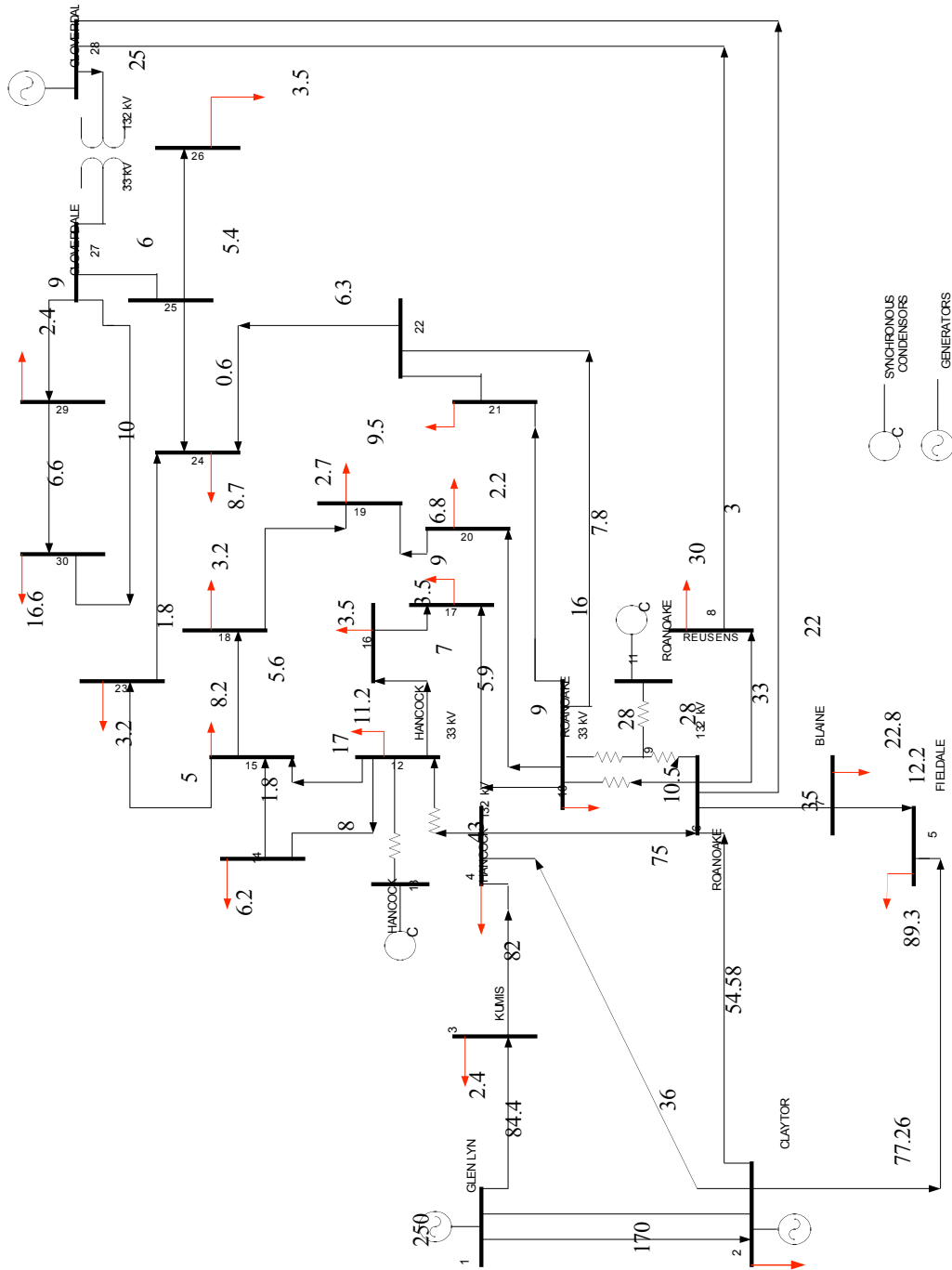


Fig. 5.5 One-line diagram of the modified IEEE 30 bus test system

The load centers in the system are designated by the arrowhead at the corresponding bus bars. The modified IEEE 30 bus test system differs from the standard IEEE 30 bus test system in having a generation center at Cloverdale (bus 28).

A central dispatch agency like an ISO may decide on the schedules for transmitting power from the utilities to the load centers in the system. Since the modified IEEE 30 bus test system has three utilities that participate in the market, there exist three transaction sets that are electronically tagged. Tables E.1 and E.2 of Appendix E depict the generation schedule and the load demand in the system during the aggregate schedule and during the transaction sets.

The wide area system under consideration is assumed to be under the effect of ten minor loop flows and the process matrix  $H$  is constructed from the network topology of the modified IEEE 30 bus test system. Fig. 5.6 portrays the paths of the minor loop flows occurring in the modified IEEE 30 bus test system. The scheduled flows during the aggregate schedule and the transaction sets are obtained as inputs from the central agency. The process matrix can be subjected to the statistical test described in section 3.3 to determine a level of confidence in estimation. Table 5.4 depicts the status of the individual test flags and the level of confidence outputted by the statistical test.

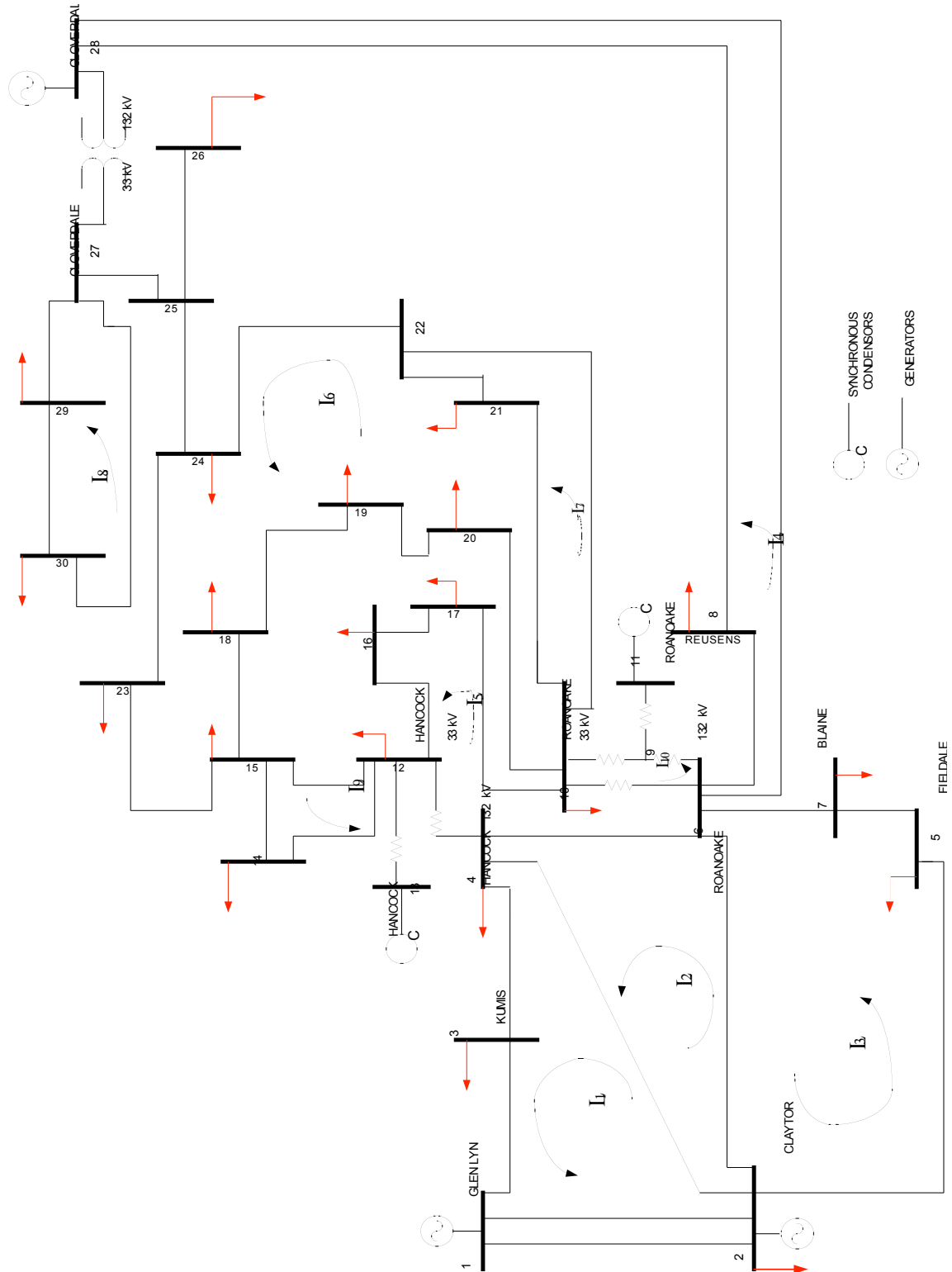


Fig. 5.6 Paths of minor loop flows in the modified IEEE 30 bus test system

TABLE 5.4

### Statistical test output for the process matrix of the modified IEEE 30 bus test systems

| Flag                | Status |
|---------------------|--------|
| VIF flag            | 0      |
| Eigenanalysis flag  | 0      |
| Rank flag           | 0      |
| Willan-Watts flag   | 0      |
| Level of confidence | HIGH   |

The statistical test indicates a '*HIGH*' level of confidence in the estimation process indicating no multicollinearity defects in the structure of the process matrix. A ridge trace of the regressors (minor loop flows) in the modified IEEE 30 bus test system depicting the variation of the values of the estimates with the bias in the system is shown in Fig. 5.7.

From the ridge trace in Fig. 5.7, it can be seen that the regressors do not change signs or show huge variation in magnitude with change in biasing parameter thus corroborating the results of the statistical test. An implication of the above results is the redundancy of ridge estimation in obtaining the minor loop flow estimates for the modified IEEE 30 bus test system. It is expected that the unbiased least squares estimator may yield similar estimates as the advanced estimation techniques. The redundancy may be gainfully employed as validation technique in the absence of other validation methods such as data splitting and gathering new data.

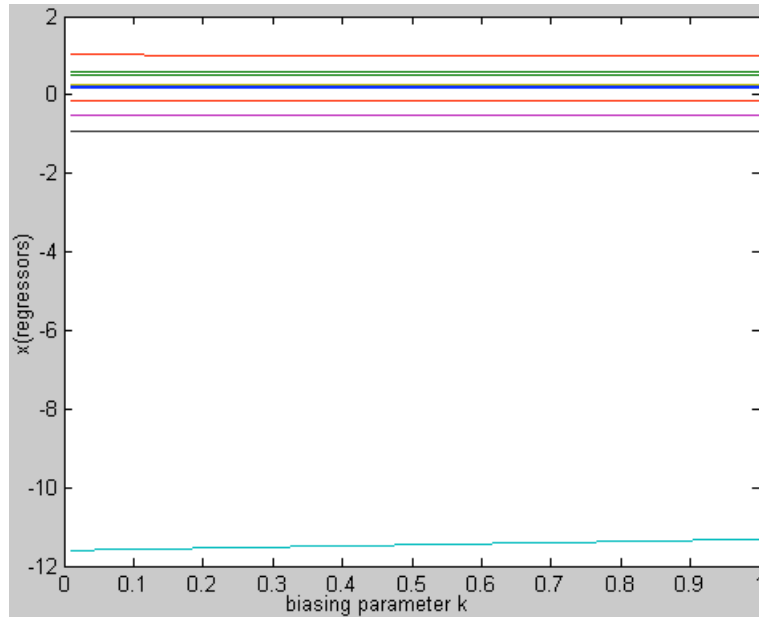


Fig. 5.7 Ridge trace of the regressors for the modified IEEE 30 bus test system

The actual flow occurring in the system during the aggregate schedule is obtained as measurements from the system while those occurring during the individual transaction sets are obtained by performing load flow studies. The USF on the transmission circuits during aggregate schedule and the individual transaction sets are obtained by the difference between the corre-

sponding actual flows and the scheduled flows. The USF corresponding to each tag is used as the measurement vector  $z$  in the linear model for the estimation process. Table E.3 represents the scheduled flow in the system during the aggregate schedule and during the transaction sets. The actual power flowing during the aggregate schedule is usually a set of noisy measurements obtained from the system. However, since the case study is performed on a test system, the observables are not readily available. Hence, simulated data sets with random white noise components in the measurements of the aggregate schedule are generated by computer programs using power flow inputs. The power flow input file for the modified IEEE 30 bus test system is given in Table E.4. Estimation of minor loop flows from noisy measurements of actual power flow can be performed using a discrete Kalman filter using a GM process as a model. The GM process is assumed to have unity variance and time constant with very large error covariance (of the order of  $10^{10}$ ). The variance of the error of the measurement process is also assumed to be unity. The discrete Kalman filter algorithm is iteratively performed to obtain noise free estimates until the relative change in the norm of the error is less than a preset threshold of 1%.

The estimates of the minor loop flows in the aggregate schedule obtained from different estimation techniques such as the least squares, ridge estimation, PCE, robust estimation, and discrete Kalman filtering can be used as validation for the model adequacy. Similarity in the estimates of the minor loop flows obtained from the different estimation techniques establishes validity of the model. Fig. 5.8 depicts a histogram plot of the estimates of the minor loop flows in the aggregate schedule of the modified IEEE 30 bus test system using different estimation techniques.

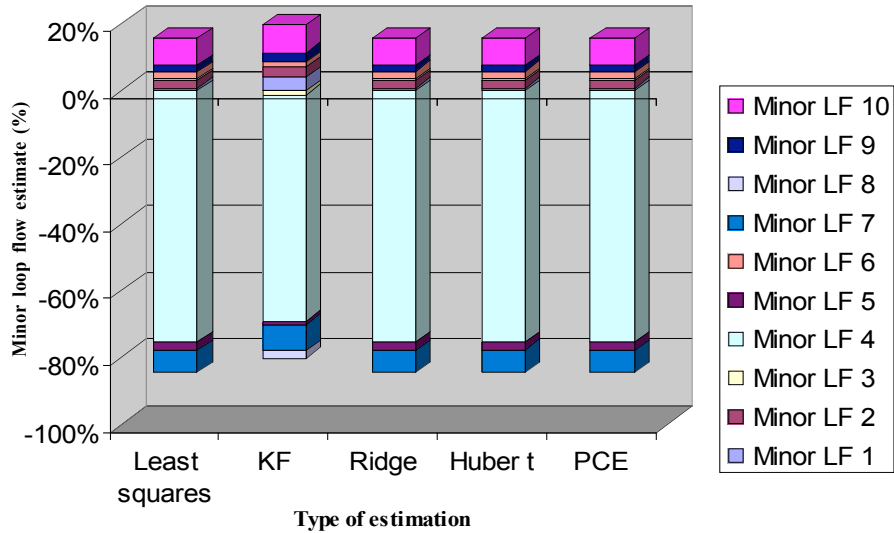


Fig. 5.8 Comparison of estimates of minor loop flows in the aggregate schedule of the modified IEEE 30 bus test system using different techniques of estimation

The minor loop flow estimates in the aggregate schedule with uncontaminated measurements are the same when estimated using the least squares method, the Huber  $t$  type robust regression, and the principal components estimation technique. The estimates of the minor loop flows in the aggregate schedule with random white noise contamination are obtained using the discrete Kalman filter. The relative error between estimates obtained using the discrete Kalman filter and the least square estimates of uncontaminated measurements is 0.0214. The relative error between estimates obtained using the unbiased least square method and the biased ridge esti-

mation is 0.0106. The low relative error between the least squares and the other estimation techniques is a motivation for employing the least squares technique for the estimation of the minor loop flows in the modified IEEE 30 bus test system. This establishes the validity of the linear model that relates the minor loop flows and the USF in the modified IEEE 30 bus test system. The same model may be employed to estimate the minor loop flows in the individual transaction sets of every GENCO in the modified IEEE 30 bus test system. The estimate of the minor loop flows in the aggregate schedule and the transaction sets of the modified IEEE 30 bus test system may now be used to determine a contribution factor for each GENCO.

The contribution factor may be determined depending upon the method of accommodation sought according to Table 5.5. The take or pay charge is designed by using the contribution factors in conjunction with an allocation type transmission pricing paradigm. In the example of the modified IEEE 30 bus test system, the distance based mile-MW method of transmission pricing at the rate of \$2 per mile per MW is employed. The difference between the market-cleared transmission price of the desired flows and the newly incurred transmission price for the actual flows in the aggregate schedule is determined. The difference in the transmission pricing introduced by the unscheduled flows can be apportioned among the GENCOs in the ratio of the individual contribution factors. A positive value of the take or pay charge represents a charge due of the GENCO toward overloading the transmission system with USF; a negative value of the take or pay charge denotes a compensation received by the GENCO for participation in the USF scenario. Table 5.5 lists the take or pay charge due of each utility in the modified IEEE 30 bus test system for a given set of schedule and transaction sets. The take or pay charge of each GENCO may be considered as signal for the respective utility to decrease loop flow in the system; this can be done by changing the generation schedule to minimize the contribution factor. Also, ISOs may adapt better scheduling techniques to reduce the total difference cost associated with the USF in the system.

TABLE 5.5

Accommodation type and corresponding take or pay charge for the GENCOs in the modified IEEE 30 bus test system

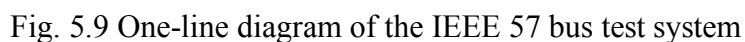
| Utility                    | Contribution factor based take or pay charge (\$) |                               |                                    |
|----------------------------|---|-------------------------------|------------------------------------|
|                            | $\ L_1\ $ based accommodation                     | $\ L_2\ $ based accommodation | $\ L_\infty\ $ based accommodation |
| 1                          | 3951.16   | 2781.03                       | 2333.38                            |
| 2                          | 464.47  | 311.58                        | 198.99                             |
| 28                         | 5739.48   | 7062.50                       | 7622.74                            |
| Total difference cost (\$) |   | 10155.11                      |                                    |

For the reasons cited in the case study on the 9 bus test system, the game theory technique based accommodation of USF is not employed. The following section describes the illustrative case study of accommodation of USF in the IEEE 57 bus test system.

#### 5.4 Case study on the IEEE 57 bus test system

The IEEE 57 bus system has 80 transmission circuits and 7 generation points as shown in the one-line diagram in Fig. 5.9.





71

Fig. 5.10 portrays the paths of the minor loop flows occurring in the IEEE 57 bus test system. The process matrix can be subjected to the statistical test described in section 3.3 to determine a level of confidence in estimation. Table 5.6 depicts the status of the individual test flags and the level of confidence outputted by the statistical test.

TABLE 5.6

Output of the statistical test for the process matrix of the IEEE 57 bus test system

| Flag                | Status |
|---------------------|--------|
| VIF flag            | 0      |
| Eigenanalysis flag  | 0      |
| Rank flag           | 0      |
| Willan-Watts flag   | 0      |
| Level of confidence | HIGH   |



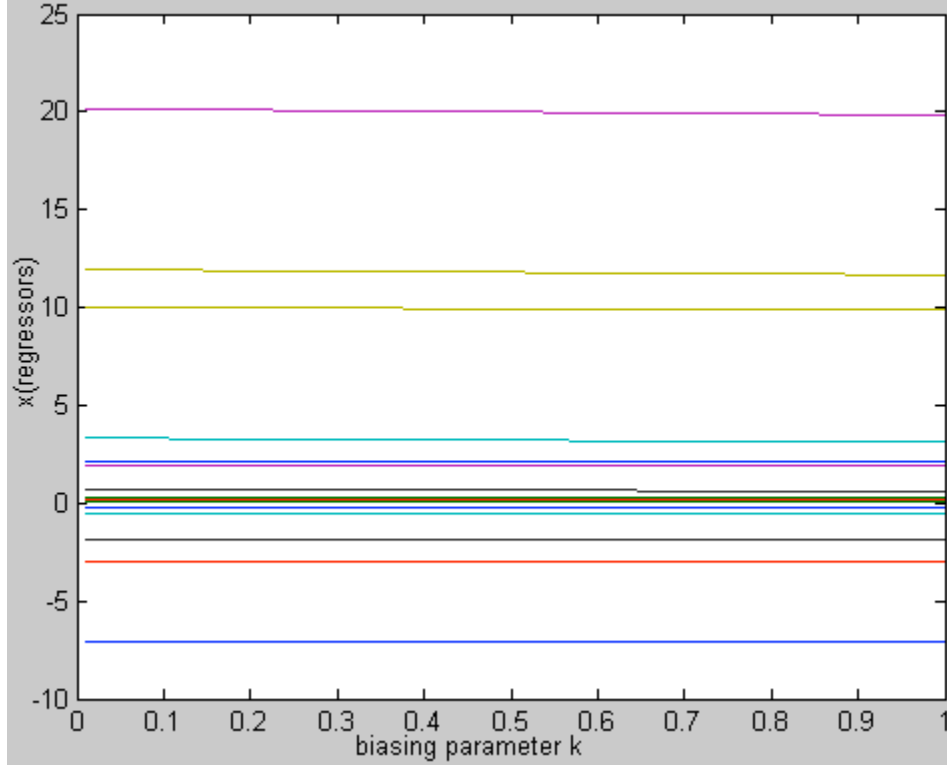


Fig. 5.11 Ridge trace of the regressors for the modified IEEE 30 bus test system

From the ridge trace in Fig. 5.11, it can be seen that the regressors do not change signs or show huge variation in magnitude with change in biasing parameter thus corroborating the results of the statistical test. An implication of the above results is the redundancy of ridge estimation in obtaining the minor loop flow estimates for the IEEE 57 bus test system. It is expected that the unbiased least squares estimator may yield similar estimates as the advanced estimation techniques. The redundancy may be gainfully employed as validation technique in the absence of other validation methods such as data splitting and gathering new data.

The actual flow occurring in the system during the aggregate schedule is obtained as measurements from the system while those occurring during the individual transaction sets are obtained by performing load flow studies. The USF on the transmission circuits during aggregate schedule and the individual transaction sets are obtained by the difference between the corresponding actual flows and the scheduled flows. The USF corresponding to each tag is used as the measurement vector  $z$  in the linear model for the estimation process. Table F.3 represents the scheduled flow in the system during the aggregate schedule and during the transaction sets. The actual power flowing during the aggregate schedule is usually a set of noisy measurements obtained from the system. However, since the case study is performed on a test system, the observables are not readily available. Hence, simulated data sets with random white noise components in the measurements of the aggregate schedule are generated by computer programs using power flow inputs. The power flow input file for the IEEE 57 bus test system is given in Table F.4.

Estimation of minor loop flows from noisy measurements of actual power flow can be performed using a discrete Kalman filter using a GM process as a model. The GM process is assumed to have unity variance and time constant with very large error covariance (of the order of  $10^{10}$ ). The variance of the error of the measurement process is also assumed to be unity. The

discrete Kalman filter algorithm is iteratively performed to obtain noise free estimates until the relative change in the norm of the error is less than a preset threshold of 1%.

The estimates of the minor loop flows in the aggregate schedule obtained from different estimation techniques such as the least squares, ridge estimation, PCE, robust estimation, and discrete Kalman filtering can be used as validation for the model adequacy. Similarity in the estimates of the minor loop flows obtained from the different estimation techniques establishes validity of the model. Fig. 5.12 depicts a histogram plot of the estimates of the minor loop flows in the aggregate schedule of the IEEE 57 bus test system using different estimation techniques.

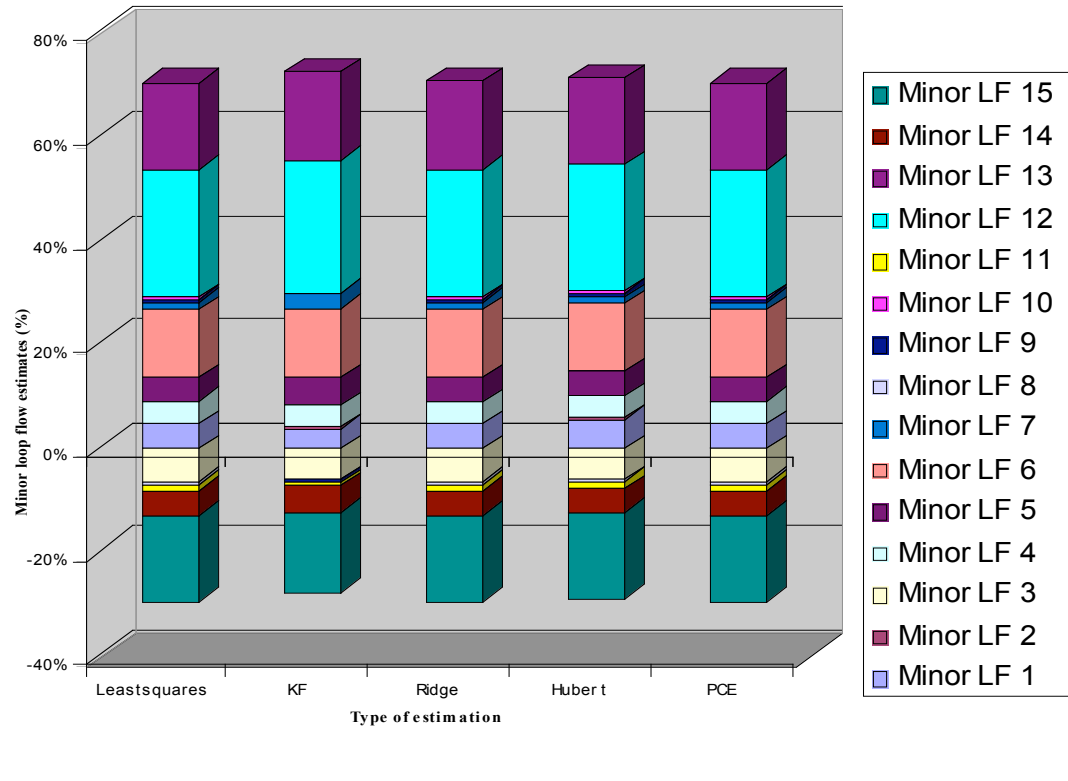


Fig. 5.12 Comparison of estimates of minor loop flows in the aggregate schedule of the IEEE 57 bus test system using different techniques of estimation

The minor loop flow estimates in the aggregate schedule with uncontaminated measurements are the same when estimated using the least squares method, the Huber  $t$  type robust regression, and the principal components estimation technique. The estimates of the minor loop flows in the aggregate schedule with random white noise contamination are obtained using the discrete Kalman filter. The relative error between estimates obtained using the discrete Kalman filter and the least square estimates of uncontaminated measurements is 0.0853. The relative error between estimates obtained using the unbiased least square method and the biased ridge estimation is 0.0034. The low relative error between the least squares and the other estimation techniques is a motivation for employing the least squares technique for the estimation of the minor loop flows in the IEEE 57 bus test system. This establishes the validity of the linear model that relates the minor loop flows and the USF in the IEEE 57 bus test system. The same model may be employed to estimate the minor loop flows in the individual transaction sets of every GENCO

in the IEEE 57 bus test system. The estimate of the minor loop flows in the aggregate schedule and the transaction sets of the IEEE 57 bus test system may now be used to determine a contribution factor for each GENCO.

The contribution factor may be determined depending upon the method of accommodation sought according to Table 5.7. The take or pay charge is designed by using the contribution factors in conjunction with an allocation type transmission pricing paradigm. In the example of the IEEE 57 bus test system, the distance based mile-MW method of transmission pricing at the rate of \$1.5 per mile per MW is employed. The difference between the market-cleared transmission price of the desired flows and the newly incurred transmission price for the actual flows in the aggregate schedule is determined. The difference in the transmission pricing introduced by the unscheduled flows can be apportioned among the GENCOs in the ratio of the individual contribution factors. A positive value of the take or pay charge represents a charge due of the GENCO toward overloading the transmission system with USF; a negative value of the take or pay charge denotes a compensation received by the GENCO for participation in the USF scenario. Table 5.7 lists the take or pay charge due of each utility in the IEEE 57 bus test system for a given set of schedule and transaction sets. The take or pay charge of each GENCO may be considered as signal for the respective utility to decrease loop flow in the system; this can be done by changing the generation schedule to minimize the contribution factor. Also, ISOs may adapt better scheduling techniques to reduce the total difference cost associated with the USF in the system.

TABLE 5.7

Accommodation type and corresponding take or pay charge for the GENCOs in the IEEE 57 bus test system

| Utility                    | Contribution factor based take or pay charge (\$) |                               |                                    |
|----------------------------|---|-------------------------------|------------------------------------|
|                            | $\ L_1\ $ based accommodation                     | $\ L_2\ $ based accommodation | $\ L_\infty\ $ based accommodation |
| 1                          | -38216.16   | -38518.52                     | -38819.80                          |
| 2                          | -7674.81  | -8575.64                      | -9861.80                           |
| 3                          | -2991.20  | -2767.71                      | -1935.35                           |
| 8                          | -14987.40   | -14334.63                     | 14077.63                           |
| 12                         | -11684.24   | -11357.31                     | -10859.23                          |
| Total difference cost (\$) |   | -75538.1                      |                                    |

For the reasons cited in the case studies on the 9 bus and the modified IEEE 30 test systems, the game theory technique based accommodation of USF is not employed. From the case studies on the 9 bus, the modified IEEE 30 bus, and the IEEE 57 test systems, the equitable accommodation of USF among GENCOs by the design of contribution factors is illustrated. The next chapter describes the development of a user friendly man machine interface prototype for performing the equitable accommodation of USF.

## CHAPTER 6

### UNSCHEDULED FLOW ACCOMMODATION SOFTWARE PROTOTYPE DESIGN

#### 6.1 An expert system to accommodate unscheduled flows

Actual flows occurring in wide area system deviate from the prearranged schedules causing unscheduled flows to occur in the system. The USF in the system may also be responsible for reduced ATC, overloading of transmission circuits, forced participation of third party players in a transaction, and uncompensated losses for the players. An accommodation of the USF among the GENCOs is described in the previous chapters to overcome the monetary deviation imposed by the USF in the system. The method has advantages of being nonselective toward players, non heuristic, transparent to the system under consideration, slack bus independent, and compatible with any allocation type transmission pricing paradigm. The method is based upon the estimation of fictitious minor loop flows, circulating in system meshes, which are assumed to cause USF in the system. The aggregate schedules and transaction sets in the system are electronically tagged and the estimates of the minor loop flows are obtained corresponding to each electronic tag. Contribution factor for each utility in the system is determined by selecting a suitable mode of accommodation based on Hölder norms. The contribution factors can be used in conjunction with any allocation type transmission pricing paradigm to determine a take or pay charge for the GENCOs for overloading the transmission system with USF. A dedicated expert system that performs all of the functions related to the accommodation of USF in a system is designed as a prototype USF management system. The expert system is a back end application that uses advanced mathematical estimators and sparse matrix power flow algorithms.

#### 6.2 USFACC- A prototype GUI for accommodation USF

An end user may not necessarily require all the information contained in the expert system for accommodating USF among GENCOs. For this reason, a user friendly menu driven GUI, the *USFACC*, that works as a front end for the USF application is designed. The GUI accepts inputs from a user pertaining to the application and interfaces with the expert system to accommodate the USF among GENCOs. The USFACC also provides results of the accommodation in an easily readable graphical form. Table 6.1 lists the item, the type, and the function performed each visualization component in the USFACC program.

TABLE 6.1

Type and function of visualization components of USFACC

| Visualization component    | Type        | Function   |
|----------------------------|-------------|--|
| System under consideration | Input data  | Enables the user to choose a type of system for accommodation of USF                                 |
| Stat test                  | Operation   | Performs the statistical test of confidence of estimation on the process matrix of the system chosen |
| VIF flag                   | Output data | Displays the Boolean flag corresponding to the VIFs of the process matrix                            |
| Rank flag                  | Output data | Displays the Boolean flag corresponding to the rank of the process matrix                            |
| Eigen flag                 | Output data | Displays the Boolean flag corresponding to the eigenvalues of the process matrix                     |
| Willan Watts flag          | Output data | Displays the Boolean flag corresponding to the Willan  |

|                                  |              |   |
|----------------------------------|--------------|---|
|                                  |              | Watts test statistic of the process matrix  |
| Level of confidence              | Output data  | Displays the level of confidence in estimation depending upon the number of Boolean flags operated      |
| Ridge trace of regressors        | Output graph | Displays a ridge trace of the regressors with respect to the biasing parameter                          |
| Type of estimation               | Input data   | Allows users to enter the technique required to estimate minor loop flows in the system                 |
| K for ridge estimator            | Input data   | Enables the user to set a biasing parameter for the ridge estimator if ridge estimation type is enabled |
| Robust estimation                | Input data   | Provides the user a variety of robust M-estimators if robust estimation type is enabled                 |
| Method of accommodation          | Input data   | Choice of accommodation technique based on Hölder norms   |
| Mile MW cost for transmission    | Input data   | Choice of a monetary value of mile MW method of transmission pricing                                    |
| Calculate                        | Operation    | Performs the estimation and accommodation of USF in the chosen system                                   |
| Contribution of utilities to USF | Output graph | Graphical display of the take or pay charge due of each utility in the USF scenario                     |
| Reset                            | Operation    | Clears the accumulator and waits for new input from user  |

Fig. 6.1 illustrates a snapshot of the USFACC screen at input. The USFACC interface screen possesses a menu bar with options of viewing the input power flow data, the output load and line flows, and a technical description of the background material. The menu bar also includes help topics pertaining to the program. The visualization components in the USFACC program are enabled with tooltips that display a text message of the corresponding functions. Fig. 6.2 depicts the snapshot of the GUI screen upon running the expert system.



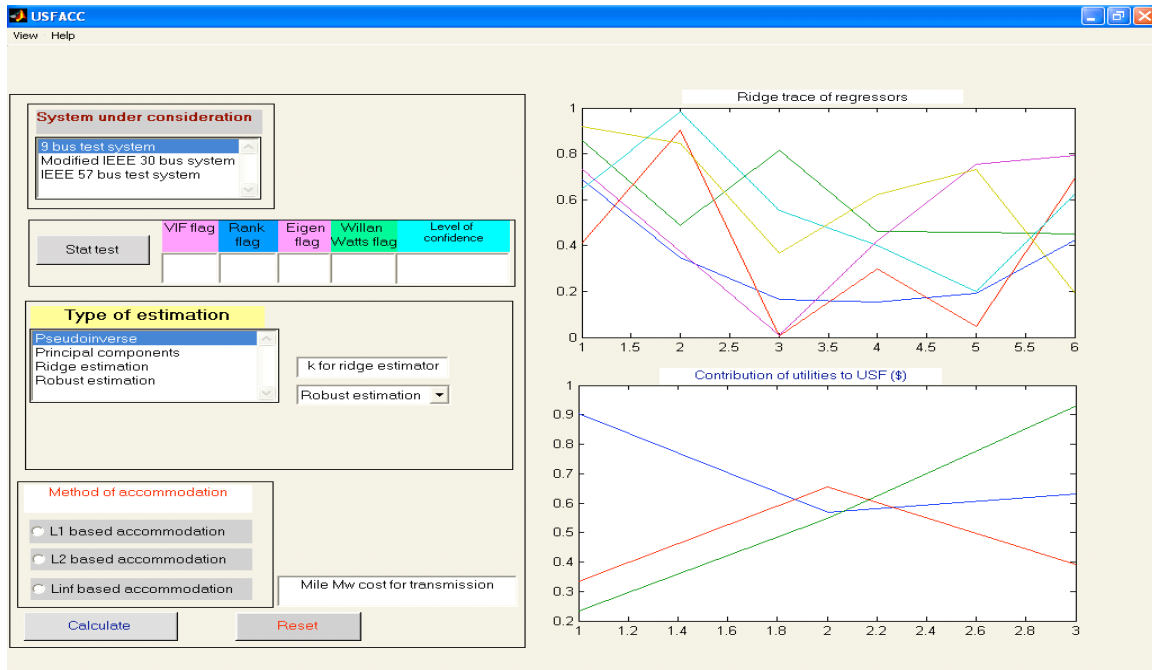


Fig. 6.1 A snapshot of the USFACC screen at input

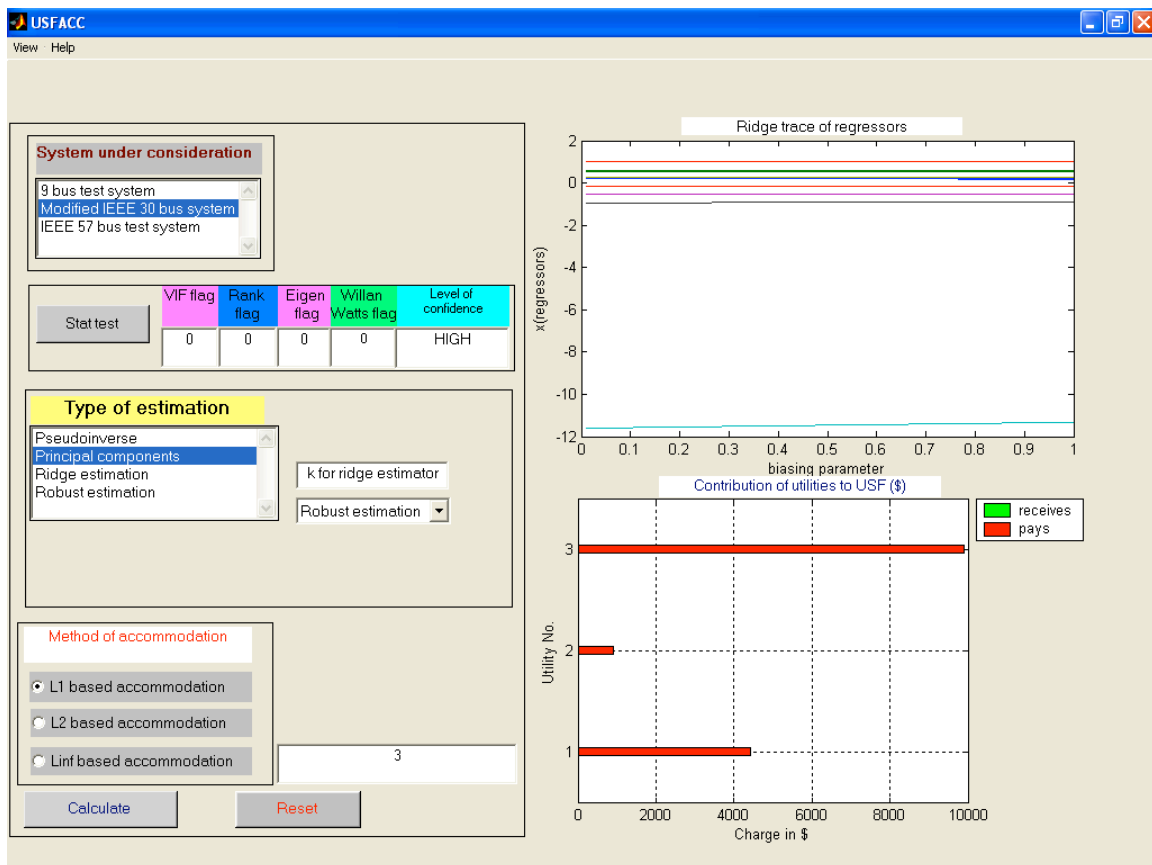


Fig. 6.2 A snapshot of the USFACC screen after performing the accommodation in a test system

The USFACC program described in this report is a beta version: an introductory version undergoing tests and modifications. The beta version of the USFACC is non portable and machine dependent as it is programmed using the MatLab software. Also, the options available to the user with respect to the types of system, estimation, and the allocation transmission pricing paradigms are limited to a few. The final working version of the USFACC is envisioned to be machine independent and portable, more user friendly with a variety of choices, and extensively menu driven.

## CHAPTER 7

### CONCLUSIONS AND RECOMMENDATIONS

#### 7.1 Conclusions

The presence of unscheduled flows in wide area systems refer to the deviation of active power flows on transmission circuits from prearranged schedules. The USF occur due to the fact that power in transmission lines flow according to physical laws than schedules. The unscheduled flows are assumed to be caused by a phenomenon called loop flows. The presence of loop flows in a system may reduce the ATC, overload lines, deviate schedules and cleared market prices, cause security and reliability related issues, force third parties to participate in power transfer without compensating the losses incurred. Hence, the loop flows, and consequently the USF, need to be accommodated in the system.

Presently the electric power industry uses several methods to control and accommodate USF among transmission companies with little success. The methods practiced have inherent problems of slack bus dependence, selectivity, inapplicability to complex systems, and empirical nature. Also, the industry accommodates the threshold level USF over selective historically qualified paths and ignores the accommodation of pre-congestion level USF on all transmission paths. Hence, a method that accommodates the pre-congestion level USF occurring on all transmission paths equitably among the GENCOs for overloading the transmission circuit is sought as a remedial measure. This method estimates fictitious minor loop flows in a wide area system that are responsible for introducing the USF. Estimation is performed using a linear model that relates the path of the minor loop flows and the USF in the system. The linear model is subjected to rigorous validation tests and simulated data is used in the absence of real data. A statistical test for establishing a level of confidence in the estimation procedure is developed in this research venture. The scope of this test is fundamental and is applicable to any large linear system model used for estimation.

Following estimation of the minor loop flows in the system, a formula for determining the contribution of each GENCO in the USF scenario is developed. Enhancements to the formula are described and several modes of accommodation are discussed depending upon the enhancement chosen.

The contribution factor generated by the formula is used in conjunction with allocation type transmission pricing paradigms for establishing a take or pay charge for each participating utility in the system. The difference in transmission pricing due to the deviation of the actual flow from the scheduled flows is apportioned in the ratio of the contribution factors to yield a take or pay charge for the GENCOs. A positive value of the charge corresponds to a penalty cost and a negative value indicates compensation to the GENCO.

The method of accommodation of USF among GENCOs possesses advantages in being nonselective, unempirical, slack bus independent, and transparent to the system under consideration.

A user friendly menu driven GUI that incorporates the tasks of modeling and estimation of minor loop flows and the accommodation of USF in a GENCO perspective is developed. A beta version is being tested and a final version is to be drafted soon.

The research on loop flow monitoring, management, near term prediction and probabilistic assessment, and prototype monitoring system design shifts the focus of the accommodation of USF from a TRANSCO perspective to a GENCO perspective.

## 7.2 Recommendations and future work

The monetary deviation from the cleared market price for transmission is a direct consequence of USF in system. The cost difference due to USF is equitably accommodated USF among GENCOs by using the contribution factors. This involves the assignment of a monetary value to each GENCO in the system for overloading the transmission circuits. A primary motivation for future research remains the minimization of the contribution factors of each utility. This venture may be performed from the GENCO perspective given the knowledge of the system topology and the accommodation technique used by the ISO.

Another potential research avenue is the probe toward better algorithms for scheduling active power flow in the wide area system. This research is envisioned to be from the ISO perspective and might involve nonlinear optimization techniques that try to match the desired flows with the actual flows in the system. An end result of this research may be the reduced levels of USF in the system.

Finally, accommodation of USF and optimization of schedules from an economic standpoint may be performed by employing powerful game theory techniques. All of the suggested future research projects are motivated toward reducing unscheduled flows in wide area networked systems and enhanced scheduling techniques.

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## APPENDIX A

### DISCRETE KALMAN FILTERING

#### A.1 Introduction to signal processing

The discrete KF algorithm is a vector state estimation method modeled on the LSM technique. The concept of Kalman filtering originated as a consequence of signal processing; to obtain system information from noise contaminated data signals. Hence to obtain a better understanding of the procedure, a short discussion on pertinent signal processing terms is required. Table A.1 defines some important terms related to signal processing.

TABLE A.1  
Definition of some terms related to signal processing

| Term                           | Definition   |
|--------------------------------|--|
| Random process                 | A non-deterministic process that can be described by functions of probabilities is called a random or stochastic process.                  |
| Gaussian random process        | A random process with the normally distributed Probability Density Functions (PDFs).   |
| Stationarity                   | Property of a random process whose density functions are time invariant.   |
| Ergodicity                     | A stochastic process whose time average is the same as the ensemble average.   |
| Autocorrelation Function (ACF) | A widely used identification/characterization measure of how well correlated a random process is with itself at two different time levels. |

Since recursive estimation deals with filtering out the noise from the measurement, a basic understanding of the structure of noise is required. White noise is the most common type of noise present in most measurement sets and is defined as a stochastic process of constant spectral density functions. The disadvantage of using white noise in modeling noise in systems is attributed to the constant amplitude feature and the associated infinite variance. A remedial technique to aid modeling is obtained by band limiting the white noise and thus making the variance finite. Bandlimited white noise has finite mean-square value and consequently, a finite variance.

There are many methods to perform recursive estimation of noisy data; however, the focus of this research work is on a particular algorithm of recursive estimation based on linear LSM called the discrete Kalman filtering.

#### A.2 Discrete Kalman filtering

Kalman filtering (KF) refers to the linear LSM based recursive processing of noisy data to yield coherent results introduced by R. E. Kalman in circa 1960 [33], [34]. The procedure is more a computer algorithm than a filter for using noisy data in state estimation. A basic requirement for this technique is a vector model of the process which is being estimated. For this purpose, the state vector of the system is expressed as,

$$\dot{x} = Fx + Gw \quad \text{and} \quad y = Bx, \quad (\text{A.4})$$

where  $x$  is a column vector of state variables,  $F$ ,  $G$ , and  $B$  are rectangular matrices of time varying elements.  $w$  is a column vector of white noise inherently associated with the process and  $y(t)$  represents the linear combination of state variables of the system. The state variables,  $x$ , of the model process, to be estimated at time instant  $t_{k+1}$  are represented in terms of the states of the system in the recent past at time instant  $t_k$ , a State Transition Matrix (STM)  $\phi_k$  at time instant  $t_k$ , and a white noise driving function  $w_k$  at time instant  $t_k$ , as ,

$$x_{k+1} = \phi_k x_k + w_k. \quad (\text{A.5})$$

The white sequence  $w_k$  is assumed to be from a set of normally distributed independent variables of zero mean and variance  $Q_k$ . The measurement process is described as,

$$Hx_k = z_k + v_k, \quad (\text{A.6})$$

where  $H$  is the rectangular process/incidence matrix,  $z_k$  is the column vector of observables or measurement, and  $v_k$  is a white noise sequence assumed to contaminate the observables at time instant  $t_k$ .  $v_k$  is assumed to be from a set of normally distributed independent variables of zero mean and variance  $R_k$  and is completely uncorrelated with  $w_k$ . The covariance matrices of  $w_k$  and  $v_k$  are represented as,

$$E[w_i w_w] = \begin{cases} Q_k, i = k \\ 0, i \neq k \end{cases}, \quad E[v_i v_w] = \begin{cases} R_k, i = k \\ 0, i \neq k \end{cases} \quad \text{and} \quad E[v_i w_w] = 0 \quad \forall i, k \quad (\text{A.7})$$

The process of recursive estimation is carried out under the assumption that the state variables at time instant  $t_k$  are known by an *a priori* knowledge of the process. The *a priori* estimate of the states at time instant  $t_k$  is represented by  $\hat{x}_k^-$ , where the super-minus indicates '*a priori*' and the hat indicates 'best estimate' [35]. The error in estimation,  $e_k^-$  and the error covariance matrix associated with the *a priori* estimate,  $P_k^-$  are given as,

$$e_k^- = x_k - \hat{x}_k^- \quad \text{and} \quad P_k^- = E[e_k^- e_k^{-T}] \quad (\text{A.8})$$

where  $x_k$  is the actual state of the system at time  $t_k$ . The best *a priori* estimate is used recursively to minimize the error by a linear blending factor called the Kalman gain. The corrected or updated state variable in each iteration is obtained by the addition of the best *a priori* estimate and the product of the Kalman gain and the measurement innovation (residual) as shown,

$$\hat{x} = \hat{x}_k^- + K_k (z - H\hat{x}_k^-), \quad (\text{A.9})$$

where  $K_k$  is the Kalman gain. The Kalman gain is designed to yield the optimal estimate by minimizing the mean-square error.

To perform the improvement of measurement and state estimation using KF algorithm, knowledge of the STM of the process,  $\phi_k$ , the process matrix  $H$ , the covariance matrices  $Q_k$  and  $R_k$ , are required. The discrete KF algorithm is begun by assuming the values of  $\hat{x}_0^-$  and  $\hat{P}_0^-$  at time instant  $t_0$ . The discrete KF algorithm contains two distinct modes viz., the measurement update and the time update steps. The Kalman gain is used as the linear blending factor to correct the best estimate of the state variable and update the error covariance matrix; this step constitutes the measurement update part of the discrete KF algorithm. This is followed by the time update step in which the *a priori* values of the state variables and the error covariance matrix are projected [35]. Table A.2 depicts the steps involved in the discrete Kalman filtering algorithm.

TABLE A.2  
Steps in the discrete Kalman filtering algorithm

| Step Number | Description of task  |
|-------------|--|
| 1           | Start iterations with <i>a priori</i> estimates of $P_k^-$ and $\hat{x}_k^-$ |
| 2           | Compute Kalman gain from $K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}$    |

|   |  |
|---|--|
| 3 | Measurement update $\hat{x} = \hat{x}_k^- + K_k(z - H\hat{x}_k^-)$<br>Error covariance update $P_k = (I - K_k H_k)P_k^-$ |
| 4 | Project ahead $\hat{x}_{k+1}^- = \phi_k \hat{x}_k$ and $P_{k+1}^- = \phi_k P_k \phi_k^T + Q_k$                           |
| 5 | Go to step 2 and keep iterating until error covariance drops below tolerance level                                       |

The above algorithm can be performed iteratively for improving the noisy measurement and estimating the states of the system in a least squares sense. The drawback of this algorithm is that the Kalman gain assumes an indeterminate form if the error covariance matrix is infinity, which is not unusual in practical cases. Hence, a better method for accommodating an infinite error covariance matrix is required. A readjustment of the terms using basic algebra in the discrete KF algorithm generates an alternative algorithm which is capable of handling large error covariance. The inversion and separation of the error covariance into two terms prevents the occurrence of the indeterminate form of the Kalman gain. The steps in the modified discrete Kalman filtering algorithm are illustrated in Table A.3. Consequently, this method can be employed in cases where the error covariance matrix is very large [35].

TABLE A.3  
Steps in the modified discrete Kalman filtering algorithm

| Step Number | Description of task  |
|-------------|--|
| 1           | Start iterations with <i>a priori</i> estimates of $(P_k^-)^{-1}$ and $\hat{x}_k^-$                          |
| 2           | Determine $P_k = ((P_k^-)^{-1} + H_k^T R_k^{-1} H_k)^{-1}$   |
| 3           | Compute Kalman gain from $K_k = P_k H_k^T R_k^{-1}$  |
| 4           | Measurement update $\hat{x} = \hat{x}_k^- + K_k(z - H\hat{x}_k^-)$   |
| 5           | Project ahead $\hat{x}_{k+1}^- = \phi_k \hat{x}_k$ and $(P_{k+1}^-)^{-1} = (\phi_k P_k \phi_k^T + Q_k)^{-1}$ |
| 6           | Go to step 2 and keep iterating until error covariance drops below tolerance level                           |

## APPENDIX B

### PRINCIPAL COMPONENTS ESTIMATION

#### B. 1 Biased estimation based on PCE

Consider a model similar to (3.1) fitted to an existing measurement data set  $z$

$$Hx = z$$

with process matrix  $H$  describing the relationship between the measurements and the estimates. The model can be expressed in a canonical form as

$$z = D\alpha, \quad (B.1)$$

where  $D = HT$ ,  $\alpha = T^T x$ ,  $T^T H^T HT = D^T D = \Lambda$ .

$\Lambda$  is a diagonal matrix of the eigenvalues  $[\lambda_1, \lambda_2, \dots, \lambda_p]$  of the matrix  $(H^T H)_{p \times p}$  and  $T_{p \times p}$  is an orthogonal matrix with columns corresponding to the eigenvectors associated with  $[\lambda_1, \lambda_2, \dots, \lambda_p]$ . A new set of orthogonal regressors called principal components are defined by the columns of  $D$ . The least squares estimates of  $\alpha$  are obtained by

$$\hat{\alpha} = (D^T D)^{-1} D^T z, \quad (B.2)$$

and the covariance matrix of  $\hat{\alpha}$  is determined as

$$\text{cov}(\hat{\alpha}) = \sigma^2 (D^T D)^{-1} = \sigma^2 \Lambda^{-1}. \quad (B.3)$$

The covariance matrix of the original estimates  $\hat{x}$  can be obtained as a linear transformation from (B.1) and (B.3) as

$$\text{cov}(\hat{x}) = \text{cov}(T\hat{\alpha}) = T\Lambda^{-1}T^T \sigma^2. \quad (B.4)$$

From (B.4), it is seen that a small value of eigenvalue, arising due to multicollinearity, may destroy the precision of the least squares estimates by inflating the variance.

The PCE method overcomes the difficulty of imprecise estimates by employing a reduced set of principal components in the model [67]. For performing the principal components estimation, the following algorithm is adhered to:

- Firstly, the eigenvalues of the  $(H^T H)$  are arranged in descending order such that  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p \geq 0$ .
- A subset containing the last  $r$  eigenvalues of  $[\lambda_1, \lambda_2, \dots, \lambda_p]$  with values close to zero is identified
- The remaining  $(p - r)$  components are selected for applying least squares to them such that

$$\hat{\alpha}_{PCE} = Y\hat{\alpha}, \quad (B.5)$$

where the first  $(p - r)$  elements of  $Y$  are unity and the remaining  $r$  elements are zero thus yielding

$$\hat{\alpha}_{PCE} = [\hat{\alpha}_1 \quad \hat{\alpha}_2 \cdots \hat{\alpha}_{p-r} \quad \hat{\alpha}_{p-r} \mid 0 \quad 0], \quad (B.6)$$

- The original estimates can be obtained by transformation using the modal matrix  $T$  such as,

$$\hat{x}_{PCE} = T\hat{\alpha}_{PCE}. \quad (B.7)$$

It should be noted that even though a reduced set of principal components are used in estimation all the original  $p$  estimates of the model are produced [67].

#### B.2. Choice of bias in PCE

A common method of determining a bias in PCE is by employing the condition indices,  $k_p$ . Condition indices are defined as the ratio of the individual eigenvalues to the maximum ei-

genvalue. A specific value of threshold depending upon the problem at hand is fixed for selecting the subset from the original principal components.

## APPENDIX C

### TRANSMISSION PRICING PARADIGMS

#### C.1 Allocation type transmission pricing paradigms

The allocation or rolled-in type transmission pricing methodologies price the users by rolling into a single value the existing as well as new transmission system costs. The rolled-in price is then allocated to each user depending upon the extent of use of the transmission system [14]. The basic types of allocation type transmission pricing paradigms include the postage stamp method, the contract path method, the distance based mile-MW method, and the power flow based mile-MW method.

- *Postage stamp method*

According to the postage stamp method, the transmission system costs are based on such a way that the distance of transmission is not a criterion for pricing. The costing is done only based on the magnitude of the power transacted. This magnitude of transacted power is measured usually during the time of system peak load condition. Equation (C.1) describes the postage stamp transmission pricing methodology.

$$C_t = (TC)(P_t)/P_{peak} , \quad (C.1)$$

where  $C_t$  is the price of transaction  $t$ ,  $TC$  is the transmission charges,  $P_t$  is the load served during transaction  $t$ , and  $P_{peak}$  is the entire system load during peak condition [14].

- *Contract path methodology*

This methodology involves the selection a particular path, called a contract path, between a supplier and a load center customer. All the transmission charges are borne by the wheeling customer. This method ignores the system operations and is not popular for accommodating the costs due to congestion or loop flows [14], [53].

- *Mile-MW method*

The mile-MW method prices transmission depending upon both the magnitude of power transmitted and the distance of transmission. This method is subdivided into two categories:

*Distance based mile-MW method*

The distance based mile-MW method prices transmission depending upon the magnitude of the transacted power and the airline distance between the points of transaction according to [14],

$$C_t = \frac{(TC)(PX_t)}{\sum PX_t} , \quad (C.2)$$

where  $PX_t$  is the mile-MW value of the transaction  $t$ .

*Power flow based mile-MW method*

The power flow based mile-MW method prices transmission according to the magnitude of the power flow and the extent of use of the transmission facility during a transaction. For this reason, this paradigm is also called *facility-to-facility* method. The cost of transmission between adjacent nodes  $m$  and  $n$  for the transaction  $t$  is,

$$C_{m,n,t} = \frac{(F_{m,n,t})(D_{m,n})(L_{m,n})}{F_{m,n}} , \quad (C.3)$$

where  $C_{m,n,t}$  is the cost in dollars for transmitting power between adjacent nodes  $m$  and  $n$  for the transaction  $t$ ,  $F_{m,n,t}$  is the amount of power in MW flowing between adjacent nodes  $m$  and  $n$  during the transaction  $t$ ,  $D_{m,n}$  is the cost in \$/MW-mile for transmitting one unit of power through one mile between adjacent nodes  $m$  and  $n$ ,  $L_{m,n}$  is the length in miles of transmission line between adjacent nodes  $m$  and  $n$ , and  $F_{m,n}$  is the total power flowing between adjacent nodes  $m$  and  $n$  taking into account all the transaction schedules [53]. If  $F_{m,n,t}$  is the only transaction between adjacent nodes  $m$  and  $n$ , then the transmission cost is given by

$$C_{m,n,t} = (F_{m,n,t})(D_{m,n})(L_{m,n}) . \quad (\text{C.4})$$

## C.2 Incremental type transmission pricing paradigms

This methodology prices the transmission incremental cost directly to a particular transaction. The incremental pricing is used in conjunction with other pricing methodologies like the composite embedded technique [14]. These incremental prices can be used with marginal costs or with run costs, either short term or long term. The marginal incremental methods price an additional unit of transaction through a linear programming method. The types of incremental transmission pricing paradigm are defined in Table C.1.

TABLE C.1  
Types of incremental transmission pricing paradigm and their definition

| Type of incremental paradigm             | Definition  |
|--|---|
| Short run incremental cost method (SRIC) | Evaluates the cost for operations associated with an additional new transaction to an existing transmission transaction. This value can be negative as it incorporates the cost for the operations incurred due to an additional transaction. |
| Long run incremental cost method (LRIC)  | Evaluates the long-run costs required to accommodate a new transaction to the existing transactions.  |
| Short run marginal cost method (SRMC)    | Marginal operating cost for a transmission transaction, which is the cost for accommodating a marginal increase in transmission, is calculated based on the Optimal Power Flow (OPF) sensitivity methods.                                     |
| Long run marginal cost method (LRMC)     | The long-run marginal costs for accommodating a marginal increase in the existing transactions are calculated.  |



## APPENDIX D

### CASE STUDY ON THE 9 BUS TEST SYSTEM

TABLE D.1

Line parameters for the branches in 9 bus test system

| Line | Resistance ( $\Omega$ ) | Reactance ( $\Omega$ ) | Length (miles) |
|------|-------------------------|------------------------|----------------|
| 1-2  | 6.0                     | 60                     | 100            |
| 1-4  | 7.2                     | 72                     | 120            |
| 2-3  | 9.6                     | 96                     | 160            |
| 2-5  | 12.0                    | 120                    | 200            |
| 3-6  | 14.4                    | 144                    | 240            |
| 4-5  | 16.8                    | 168                    | 280            |
| 4-7  | 6.6                     | 66                     | 110            |
| 5-6  | 9.0                     | 90                     | 150            |
| 5-8  | 18.0                    | 180                    | 300            |
| 6-9  | 10.5                    | 105                    | 175            |
| 7-8  | 10.8                    | 108                    | 180            |
| 8-9  | 12.0                    | 120                    | 200            |

TABLE D.2

Generation schedule during aggregate schedule in the 9 bus test system

| Bus No.         | 1   | 3   | 7   |
|-----------------|-----|-----|-----|
| Generation (MW) | 100 | 200 | 200 |

TABLE D.3

Load demand during aggregate schedule in the 9 bus test system

| Bus No.          | 2  | 4  | 5   | 8   | 9   |
|------------------|----|----|-----|-----|-----|
| Load served (MW) | 75 | 75 | 125 | 100 | 125 |

TABLE D.4

Desired flows in transmission circuits during the aggregate schedule in the 9 bus test system

| Branch<br>From-To    | 1-2 | 1-4 | 2-3 | 2-5 | 3-6 | 4-5 | 4-7 | 5-6 | 5-8 | 6-9 | 7-8 | 8-9 |
|----------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Desired flow<br>(MW) | 85  | 15  | -50 | 60  | 150 | -35 | -25 | -85 | -15 | 65  | 175 | 60  |

TABLE D.5

Generation and corresponding load schedules in the 9 bus test system

|             | Generator 1 (MW) | Generator 3 (MW) | Generator 7 (MW) |
|-------------|------------------|------------------|------------------|
| Load 2 (MW) | 25               | 50               | 0                |
| Load 4 (MW) | 15               | 35               | 25               |
| Load 5 (MW) | 50               | 95               | 25               |
| Load 8 (MW) | 10               | 25               | 65               |
| Load 9 (MW) | 0                | 40               | 85               |

TABLE D.6

Actual flows occurring in 9 bus system during aggregate and tagged schedules

| Branch                           | 1-2   | 1-4  | 2-3   | 2-5  | 3-6   | 4-5  | 4-7  | 5-6  | 5-8  | 6-9  | 7-8    | 8-9   |
|----------------------------------|-------|------|-------|------|-------|------|------|------|------|------|--------|-------|
| Aggregate schedule with WGN (MW) | 57.87 | -30  | -92.8 | 69.3 | 98.75 | -115 | 11.9 | -143 | 7.07 | 86.5 | 217.99 | 74.03 |
| Tagged schedule 1 (MW)           | 56.6  | 44.7 | 8.7   | 22.6 | 8.7   | 16.7 | 12.8 | 7.9  | -3.2 | 0.5  | 12.7   | -0.5  |
| Tagged schedule 2 (MW)           | 33.6  | -30  | 17.5  | 16   | 17.5  | 34.5 | -89  | 22.1 | 2.5  | 39.2 | 110    | 46.3  |
| Tagged schedule 3 (MW)           | 35.6  | 39.1 | -118  | 32.1 | 81    | 4.5  | 8.4  | -35  | 11.8 | 45.1 | 8.3    | -4.9  |

## APPENDIX E

### CASE STUDY ON THE MODIFIED IEEE 30 BUS TEST SYSTEM

TABLE E.1

Schedule of generation in the modified IEEE 30 bus test system

| Bus name (number) | Generation (MW) |
|-------------------|-----------------|
| Glen Lyn (1)      | 260             |
| Claytor (2)       | 40              |
| Cloverdale (28)   | 100             |

TABLE E.2

Load schedule during aggregate schedule and tagged schedules in the modified IEEE 30 bus test system

| Bus number | Load (MW)          |                |                |                |
|------------|--------------------|----------------|----------------|----------------|
|            | Aggregate schedule | Schedule tag 1 | Schedule tag 2 | Schedule tag 3 |
| 2          | 21.7               | -              | 21.7           | -              |
| 3          | 2.4                | 2.4            | -              | -              |
| 4          | 7.6                | -              | 7.6            | -              |
| 5          | 94.2               | 89.3           | 4.9            | -              |
| 7          | 22.8               | 22.8           | -              | -              |
| 8          | 90                 | 30             | -              | 60             |
| 10         | 5.8                | -              | 5.8            | -              |
| 12         | 11.2               | 11.2           | -              | -              |
| 14         | 6.2                | 6.2            | -              | -              |
| 15         | 8.2                | 8.2            | -              | -              |
| 16         | 3.5                | 3.5            | -              | -              |
| 17         | 9                  | 9              | -              | -              |
| 18         | 3.2                | 3.2            | -              | -              |
| 19         | 9.5                | 9.5            | -              | -              |
| 20         | 2.2                | 2.2            | -              | -              |
| 21         | 17.5               | 17.5           | -              | -              |
| 23         | 3.2                | 3.2            | -              | -              |
| 24         | 33.7               | 8.7            | -              | 25             |
| 26         | 18.5               | 3.5            | -              | 15             |
| 29         | 2.4                | 2.4            | -              | -              |
| 30         | 10.6               | 10.6           | -              | -              |

TABLE E.3

Actual and scheduled power flows occurring in the modified IEEE 30 bus test system during aggregate schedule and tagged schedules

| Branch | Aggregate schedule | Schedule tag 1 | Schedule tag 2 | Schedule tag 3 |
|--------|--------------------|----------------|----------------|----------------|
|--------|--------------------|----------------|----------------|----------------|

|       | Actual<br>(MW) | Scheduled<br>(MW) | Actual<br>(MW) | Scheduled<br>(MW) | Actual<br>(MW) | Scheduled<br>(MW) | Actual<br>(MW) | Scheduled<br>(MW) |
|-------|----------------|-------------------|----------------|-------------------|----------------|-------------------|----------------|-------------------|
| 1-2   | 173.5          | 172.9             | 175.8322       | 170               | -3.4521        | 0                 | -0.2783        | 0                 |
| 1-3   | 82.5           | 87.9              | 84.0276        | 84.4              | 3.4521         | 0                 | 0.2783         | 0                 |
| 2-4   | 45.8           | 43.52             | 38.1779        | 36                | 5.6500         | 7.6               | 1.0545         | 0                 |
| 3-4   | 85.6           | 82.43             | 78.7844        | 82                | 3.3682         | 0                 | 0.1842         | 0                 |
| 2-5   | 77.3           | 82.16             | 77.1173        | 77.26             | 4.3871         | 4.9               | -0.9099        | 0                 |
| 2-6   | 66.03          | 60.38             | 55.1688        | 54.58             | 5.6830         | 5.8               | -0.5547        | 0                 |
| 4-6   | 72.68          | 73.53             | 73.8971        | 75                | -0.4070        | 0                 | -8.0359        | 0                 |
| 5-7   | -14.85         | -14.96            | -14.7664       | -12.2             | -0.6520        | 0                 | -1.0612        | 0                 |
| 6-7   | 39.5           | 38.29             | 38.0928        | 35                | 0.6799         | 0                 | 1.0860         | 0                 |
| 6-8   | 66.5           | 29.49             | 29.4344        | 33                | 0.0334         | 0                 | 35.2049        | 0                 |
| 6-9   | 38.44          | 28.7              | 26.3248        | 28                | 2.9967         | 0                 | 10.6308        | 0                 |
| 6-10  | 25.34          | 16.2              | 14.8           | 10.5              | 1.6917         | 0                 | 5.9984         | 0                 |
| 9-11  | -2.63          | 0                 | 0              | 0                 | 0              | 0                 | 0              | 0                 |
| 9-10  | 49.07          | 28.7              | 26.324         | 28                | 2.9967         | 0                 | 10.6308        | 0                 |
| 4-12  | 52.09          | 42.9              | 41.5196        | 43                | 1.7551         | 0                 | 9.2020         | 0                 |
| 12-13 | 0.511          | 0                 | 0              | 0                 | 0              | 0                 | 0              | 0                 |
| 12-14 | 14.24          | 7.71              | 7.5943         | 8                 | 0.3128         | 0                 | 1.7600         | 0                 |
| 12-15 | 24.27          | 17.3              | 16.8546        | 17                | 0.6746         | 0                 | 6.2800         | 0                 |
| 12-16 | 7.61           | 6.67              | 5.8707         | 7                 | 0.7677         | 0                 | 1.1620         | 0                 |
| 14-15 | -0.6           | 1.44              | 1.3211         | 1.8               | 0.3058         | 0                 | 1.7498         | 0                 |
| 16-17 | 5.8            | 3.1               | 2.3331         | 3.5               | 0.7592         | 0                 | 1.1524         | 0                 |
| 15-18 | -0.70          | 5.66              | 5.2077         | 5.6               | 0.4963         | 0                 | -0.3787        | 0                 |
| 18-19 | 5.26           | 2.42              | 1.9775         | 2.7               | 0.4936         | 0                 | -0.3809        | 0                 |
| 19-20 | -0.1606        | -7.0              | -7.5250        | -6.8              | 0.4932         | 0                 | -0.3811        | 0                 |
| 10-20 | 6.67           | 9.4               | 9.8473         | 9                 | -0.4800        | 0                 | 0.3956         | 0                 |
| 10-17 | 8.39           | 5.9               | 6.6912         | 5.9               | -0.7516        | 0                 | -1.1443        | 0                 |

TABLE E.3

Actual and scheduled power flows occurring in the modified IEEE 30 bus test system during aggregate schedule and tagged schedules (continued)

| Branch | Aggregate schedule | Schedule tag 1 | Schedule tag 2 | Schedule tag 3 |
|--------|--------------------|----------------|----------------|----------------|
|--------|--------------------|----------------|----------------|----------------|

|       | Actual<br>(MW) | Scheduled<br>(MW) | Actual<br>(MW) | Scheduled<br>(MW) | Actual<br>(MW) | Scheduled<br>(MW) | Actual<br>(MW) | Scheduled<br>(MW) |
|-------|----------------|-------------------|----------------|-------------------|----------------|-------------------|----------------|-------------------|
| 10-21 | 32.2           | 16                | 16.5180        | 16                | 0.0231         | 0                 | 10.4505        | 0                 |
| 10-22 | 7.52           | 7.8               | 8.0995         | 7.8               | 0.0968         | 0                 | 6.9274         | 0                 |
| 21-22 | 2.32           | -1.5              | -1.1094        | -1.5              | -0.0135        | 0                 | 10.3750        | 0                 |
| 15-23 | 15.8           | 4.7               | 4.5596         | 5                 | 0.4552         | 0                 | 8.3426         | 0                 |
| 22-24 | 21.5           | 6.17              | 6.9274         | 6.3               | 0.0652         | 0                 | 17.2382        | 0                 |
| 23-24 | 13             | 1.52              | 1.3298         | 1.8               | 0.4445         | 0                 | 8.2594         | 0                 |
| 24-25 | 2.3            | -26               | -0.5198        | -0.6              | 0.4843         | 0                 | 0.0462         | -25               |
| 25-26 | 22.8           | 18.5              | 3.5472         | 5.4               | 0.0135         | 0                 | 15.6836        | 15                |
| 25-27 | -15.2          | -44.6             | -4.0843        | -6                | 0.4429         | 0                 | -15.668        | -40               |
| 28-27 | 37.8           | 57.9              | 17.4038        | 25                | -0.4289        | 0                 | 15.9625        | 40                |
| 27-29 | 11.54          | 6.2               | 6.1981         | 9                 | 0.0046         | 0                 | 0.0047         | 0                 |
| 27-30 | 1.71           | 7.1               | 7.1024         | 10                | 0.0067         | 0                 | 0.0068         | 0                 |
| 29-30 | 3.61           | 3.7               | 3.7062         | 6.6               | 0.0001         | 0                 | 0.0001         | 0                 |
| 8-28  | -25.51         | -60.6             | -0.6671        | 3                 | -0.0356        | 0                 | -24.95         | -60               |
| 6-28  | -46.7          | 18.63             | 18.1353        | 22                | -0.3107        | 0                 | -61.74         | 0                 |

TABLE E.4

Sample power flow input file for the modified IEEE 30 bus test system

```

AMERICAN ELECTRIC POWER 30 BUS SYSTEM
PARAMETER DATA
-999
BUS DATA
1 GLEN LYN 132 1 0 3 0.0000 0.00 0.00 0.00 260.00 0.00
  1.0600 0.00 0.00 0.0000 0.0000 0
2 CLAYTOR 132 1 0 2 0.0000 0.00 21.70 12.70 40.00 0.00
  1.0450 60.00 -40.00 0.0000 0.0000 0
3 KUMIS 132 1 0 0 0.0000 0.00 2.40 1.20 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
4 HANCOCK 132 1 0 0 0.0000 0.00 7.60 1.60 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
5 FIELDALE 132 1 0 2 0.0000 0.00 94.20 19.00 0.00 0.00
  1.0100 40.00 -40.00 0.0000 0.0000 0
6 ROANOKE 132 1 0 0 0.0000 0.00 0.00 0.00 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
7 BLAINE 132 1 0 0 0.0000 0.00 22.80 10.90 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
8 REUSENS 132 1 0 2 0.0000 0.00 30.00 30.00 0.00 0.00
  1.0100 40.00 -10.00 0.0000 0.0000 0
9 ROANOKE 3WT 1 0 0 0.0000 0.00 0.00 0.00 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
10 ROANOKE 33 3 0 0 0.0000 0.00 5.80 2.00 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.1900 0
11 ROANOKE SCAP 1 0 2 0.0000 0.00 0.00 0.00 0.00 0.00
  1.0820 24.00 -6.00 0.0000 0.0000 0
12 HANCOCK 33 3 0 0 0.0000 0.00 11.20 7.50 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
13 HANCOCK SCAP 3 0 2 0.0000 0.00 0.00 0.00 0.00 0.00
  1.0710 24.00 -6.00 0.0000 0.0000 0
14 LOAD14 33 3 0 0 0.0000 0.00 6.20 1.60 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
15 LOAD15 33 3 0 0 0.0000 0.00 8.20 2.50 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
16 LOAD16 33 3 0 0 0.0000 0.00 3.50 1.80 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
17 LOAD17 33 3 0 0 0.0000 0.00 9.00 5.80 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
18 LOAD18 33 3 0 0 0.0000 0.00 3.20 0.90 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
19 LOAD19 33 3 0 0 0.0000 0.00 9.50 3.40 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
20 LOAD20 33 3 0 0 0.0000 0.00 2.20 0.70 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
21 LOAD21 33 3 0 0 0.0000 0.00 17.50 11.20 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
22 JUNCTN22 33 3 0 0 0.0000 0.00 0.00 0.00 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
23 LOAD23 33 2 0 0 0.0000 0.00 3.20 1.60 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
24 LOAD24CAP 33 2 0 0 0.0000 0.00 8.70 6.70 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0430 0
25 JUNCTN25 33 2 0 0 0.0000 0.00 0.00 0.00 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0
26 LOAD26 33 2 0 0 0.0000 0.00 3.50 2.30 0.00 0.00
  0.0000 0.00 0.00 0.0000 0.0000 0

```

```

27 CLOVERDALE33 2 0 0 0.0000 0.00 0.00 0.00 0.00 0.00
0.0000 0.00 0.00 0.0000 0.0000 0
28 CLOVERDALI32 2 0 0 0.0000 0.00 0.00 0.00 0.00 0.00
0.0000 0.00 0.00 0.0000 0.0000 0
29 LOAD29 33 2 0 0 0.0000 0.00 2.40 0.90 0.00 0.00
0.0000 0.00 0.00 0.0000 0.0000 0
30 LOAD30 33 2 0 0 0.0000 0.00 10.60 1.90 0.00 0.00
0.0000 0.00 0.00 0.0000 0.0000 0

```

-999

#### BRANCH DATA

```

1 2 1 0 1 0 0.019200 0.057500 0.05280 0 0 0 0 0
1 3 1 0 1 0 0.045200 0.185200 0.04080 0 0 0 0 0
2 4 1 0 1 0 0.057000 0.173700 0.03680 0 0 0 0 0
3 4 1 0 1 0 0.013200 0.037900 0.00840 0 0 0 0 0
2 5 1 0 1 0 0.047200 0.198300 0.04180 0 0 0 0 0
2 6 1 0 1 0 0.058100 0.176300 0.03740 0 0 0 0 0
4 6 1 0 1 0 0.011900 0.041400 0.00900 0 0 0 0 0
5 7 1 0 1 0 0.046000 0.116000 0.02040 0 0 0 0 0
6 7 1 0 1 0 0.026700 0.082000 0.01700 0 0 0 0 0
6 8 1 0 1 0 0.012000 0.042000 0.00900 0 0 0 0 0
6 9 1 0 1 1 0.000000 0.208000 0.00000 0 0 0 0 0
0.9780 0.00 0.0000 0.00000.00000 0.0000 0.0000
6 10 1 0 1 1 0.000000 0.556000 0.00000 0 0 0 0 0
0.9690 0.00 0.0000 0.00000.00000 0.0000 0.0000
9 11 1 0 1 0 0.000000 0.208000 0.00000 0 0 0 0 0
9 10 1 0 1 0 0.000000 0.110000 0.00000 0 0 0 0 0
4 12 1 0 1 1 0.000000 0.256000 0.00000 0 0 0 0 0
0.9320 0.00 0.0000 0.00000.00000 0.0000 0.0000
12 13 3 0 1 0 0.000000 0.140000 0.00000 0 0 0 0 0
12 14 3 0 1 0 0.123100 0.255900 0.00000 0 0 0 0 0
12 15 3 0 1 0 0.066200 0.130400 0.00000 0 0 0 0 0
12 16 3 0 1 0 0.094500 0.198700 0.00000 0 0 0 0 0
14 15 3 0 1 0 0.221000 0.199700 0.00000 0 0 0 0 0
16 17 3 0 1 0 0.082400 0.192300 0.00000 0 0 0 0 0
15 18 3 0 1 0 0.107300 0.218500 0.00000 0 0 0 0 0
18 19 3 0 1 0 0.063900 0.129200 0.00000 0 0 0 0 0
19 20 3 0 1 0 0.034000 0.068000 0.00000 0 0 0 0 0
10 20 3 0 1 0 0.093600 0.209000 0.00000 0 0 0 0 0
10 17 3 0 1 0 0.032400 0.084500 0.00000 0 0 0 0 0
10 21 3 0 1 0 0.034800 0.074900 0.00000 0 0 0 0 0
10 22 3 0 1 0 0.072700 0.149900 0.00000 0 0 0 0 0
21 22 3 0 1 0 0.023200 0.047200 0.00000 0 0 0 0 0
21 22 3 0 2 0 0.023200 0.047200 0.00000 0 0 0 0 0
15 23 3 0 1 0 0.100000 0.202000 0.00000 0 0 0 0 0
22 24 2 0 1 0 0.115000 0.179000 0.00000 0 0 0 0 0
23 24 2 0 1 0 0.132000 0.270000 0.00000 0 0 0 0 0
24 25 2 0 1 0 0.188500 0.329200 0.00000 0 0 0 0 0
25 26 2 0 1 0 0.254400 0.380000 0.00000 0 0 0 0 0
25 27 2 0 1 0 0.109300 0.208700 0.00000 0 0 0 0 0
27 28 2 0 1 1 0.000000 0.792000 0.00000 0 0 0 0 0
1.0330 0.00 0.0000 0.00000.00000 0.0000 0.0000
27 28 2 0 2 1 0.000000 0.792000 0.00000 0 0 0 0 0
1.0330 0.00 0.0000 0.00000.00000 0.0000 0.0000
27 29 2 0 1 0 0.219800 0.415300 0.00000 0 0 0 0 0
27 30 2 0 1 0 0.320200 0.602700 0.00000 0 0 0 0 0
29 30 2 0 1 0 0.239900 0.453300 0.00000 0 0 0 0 0
8 28 2 0 1 0 0.063600 0.200000 0.04280 0 0 0 0 0
6 28 2 0 1 0 0.050700 0.179700 0.00433 0 0 0 0 0
6 28 2 0 2 0 0.050700 0.179700 0.00433 0 0 0 0 0
6 28 2 0 3 0 0.050700 0.179700 0.00433 0 0 0 0 0

```

-999

## APPENDIX F

### CASE STUDY ON THE IEEE 57 BUS TEST SYSTEM

TABLE F.1

Generation schedule in the IEEE 57 bus system

| Bus name (number) | Amos (1) | Baker (2) | Crawford (3) | Grange (8) | Loescher (12) |
|-------------------|----------|-----------|--------------|------------|---------------|
| Generation (MW)   | 690      | 200       | 100          | 450        | 275           |

TABLE F.2

Load schedule in the IEEE 57 bus system during tagged schedules and aggregate schedule

| Bus name (number) | Load (MW)      |                |                |                |                |                    |
|-------------------|----------------|----------------|----------------|----------------|----------------|--------------------|
|                   | Schedule tag 1 | Schedule tag 2 | Schedule tag 3 | Schedule tag 4 | Schedule tag 5 | Aggregate schedule |
| Amos (1)          | -              | 23             | -              | -              | -              | 23                 |
| Baker (2)         | -              | 3              | -              | -              | -              | 3                  |
| Crawford (3)      | -              | 1              | 100            | -              | -              | 101                |
| Doyle (4)         | -              | -              | -              | -              | -              | -                  |
| Dawson (5)        | -              | 13             | -              | -              | -              | 13                 |
| Ells (6)          | -              | 75             | -              | -              | -              | 75                 |
| Farlie (7)        | -              | -              | -              | -              | -              | -                  |
| Grange (8)        | -              | -              | -              | 150            | -              | 150                |
| Homer (9)         | 12             | -              | -              | 109            | -              | 121                |
| Jenkins (10)      | 32             | -              | -              | -              | -              | 32                 |
| Kincaid (11)      | 61             | 11             | -              | 32             | -              | 100                |
| Loescher (12)     | 102            | -              | -              | -              | 275            | 377                |
| Moses (13)        | 18             | -              | -              | -              | -              | 18                 |
| Nestle (14)       | 10.5           | -              | -              | -              | -              | 10.5               |
| Oakdale (15)      | 16             | 6              | -              | -              | -              | 22                 |
| Andy (16)         | 43             | -              | -              | -              | -              | 43                 |
| Andy (17)         | 40             | 2              | -              | -              | -              | 42                 |
| Doyle (18)        | -              | 10             | -              | 17.2           | -              | 27.2               |
| Richter (19)      | -              | -              | -              | 3.3            | -              | 3.3                |
| Bus name (number) | Load (MW)      |                |                |                |                |                    |
|                   | Schedule tag 1 | Schedule tag 2 | Schedule tag 3 | Schedule tag 4 | Schedule tag 5 | Aggregate schedule |
| Richter (20)      | -              | -              | -              | 2.3            | -              | 2.3                |
| Beaver2 (22)      | -              | -              | -              | -              | -              | -                  |
| Beaver1 (23)      | 6.3            | -              | -              | -              | -              | 6.3                |



|                |     |     |   |     |   |      |
|----------------|-----|-----|---|-----|---|------|
| Pool (24)      | -   | -   | - | -   | - | -    |
| Pool A (25)    | 6.3 | -   | - | -   | - | 6.3  |
| Pool (26)      | -   | -   | - | -   | - | -    |
| Hamel (27)     | -   | -   | - | 9.3 | - | 9.3  |
| Wyncote (28)   | -   | -   | - | 4.6 | - | 4.6  |
| Farlie (29)    | -   | 2   | - | 50  | - | 52   |
| Chester (30)   | 3.6 | -   | - | -   | - | 3.6  |
| Hanover (31)   | 5.6 | -   | - | -   | - | 5.8  |
| Uxbridge (32)  | 14  | -   | - | -   | - | 14   |
| Luxbridge (33) | 3.8 | -   | - | -   | - | 3.8  |
| Uxbridge (34)  | -   | -   | - | -   | - | -    |
| Copely MN (35) | 6   | -   | - | -   | - | 6    |
| Copely (36)    | -   | -   | - | -   | - | -    |
| N Copely (37)  | -   | -   | - | -   | - | -    |
| Stanton (38)   | -   | 14  | - | -   | - | 14   |
| W Taunton (39) | -   | -   | - | -   | - | -    |
| Taunton (40)   | -   | -   | - | -   | - | -    |
| Kincaid (41)   | 1.3 | 5   | - | -   | - | 6.3  |
| Airport (42)   | -   | 7.1 | - | -   | - | 7.1  |
| Kincaid (43)   | -   | 2   | - | -   | - | 2    |
| S Oakdale (44) | -   | 12  | - | -   | - | 12   |
| Oakdale (45)   | -   | -   | - | -   | - | -    |
| Nestle (46)    | -   | -   | - | -   | - | -    |
| Airport 2 (47) | 30  | -   | - | -   | - | 29.7 |
| Airport 1 (48) | -   | -   | - | -   | - | -    |
| Moses (49)     | 18  | -   | - | -   | - | 18   |
| Manx (50)      | -   | -   | - | 21  | - | 21   |
| Jenkins (51)   | 18  | -   | - | -   | - | 18   |
| N Vexley (52)  | -   | -   | - | 4.9 | - | 4.9  |
| Vexley (53)    | -   | -   | - | 30  | - | 30   |
| Vexley SQ (54) | -   | -   | - | 4.1 | - | 4.1  |
| Homer (55)     | -   | -   | - | 6.8 | - | 6.8  |
| Taunton (56)   | -   | 7.6 | - | -   | - | 7.6  |
| W Taunton (57) | -   | 6.7 | - | -   | - | 6.7  |

TABLE F.3

Desired flows in the IEEE 57 bus system during tagged schedules and aggregate schedule

| From bus | To bus | Line flow (MW)     |                |                |                |                |                 |
|----------|--------|--------------------|----------------|----------------|----------------|----------------|-----------------|
|          |        | Aggregate Schedule | Schedule tag 1 | Schedule tag 2 | Schedule tag 3 | Schedule tag 8 | Schedule tag 12 |
| 1        | 2      | -25                | 0              | -25            | 0              | 0              | 0               |
| 1        | 15     | 430                | 430            | 0              | 0              | 0              | 0               |
| 1        | 16     | 120                | 120            | 0              | 0              | 0              | 0               |
| 1        | 17     | 142                | 140            | 2              | 0              | 0              | 0               |
| 2        | 3      | 172                | 0              | 172            | 0              | 0              | 0               |
| 3        | 4      | 100                | 0              | 100            | 0              | 0              | 0               |
| 3        | 15     | 71                 | 0              | 71             | 0              | 0              | 0               |
| 4        | 5      | 40                 | 0              | 40             | 0              | 0              | 0               |
| 4        | 6      | 50                 | 0              | 50             | 0              | 0              | 0               |
| 4        | 18     | 5                  | 0              | 5              | 0              | 0              | 0               |
| 4        | 18     | 5                  | 0              | 5              | 0              | 0              | 0               |
| 5        | 6      | 27                 | 0              | 27             | 0              | 0              | 0               |
| 6        | 7      | 32                 | 0              | 2              | 0              | 30             | 0               |
| 6        | 8      | -30                | 0              | 0              | 0              | -30            | 0               |
| 7        | 8      | -85                | 0              | 0              | 0              | -85            | 0               |
| 7        | 29     | 117                | 0              | 2              | 0              | 115            | 0               |
| 8        | 9      | 185                | 0              | 0              | 0              | 185            | 0               |
| 9        | 10     | 51                 | 0              | 0              | 0              | 51             | 0               |
| 9        | 11     | 0                  | 0              | 0              | 0              | 0              | 0               |
| 9        | 12     | -12                | -12            | 0              | 0              | 0              | 0               |
| 9        | 13     | -20                | 0              | 0              | 0              | 0              | 0               |
| 9        | 55     | 25                 | 0              | 0              | 0              | 25             | 0               |
| 10       | 12     | -50                | -50            | 0              | 0              | 0              | 0               |
| 10       | 51     | 67                 | 18             | 0              | 0              | 51             | 0               |
| From bus | To bus | Line flow (MW)     |                |                |                |                |                 |
|          |        | Aggregate Schedule | Schedule tag 1 | Schedule tag 2 | Schedule tag 3 | Schedule tag 8 | Schedule tag 12 |
| 11       | 41     | -30                | -10.8          | 6              | 0              | 16             | 0               |
| 11       | 43     | -28                | -10.8          | 5              | 0              | 16             | 0               |
| 12       | 13     | 12                 | 12             | 0              | 0              | 0              | 0               |
| 12       | 16     | -77                | -77            | 0              | 0              | 0              | 0               |
| 12       | 17     | -100               | -100           | 0              | 0              | 0              | 0               |
| 13       | 14     | -60                | -60            | 0              | 0              | 0              | 0               |
| 13       | 15     | -188               | -188           | 0              | 0              | 0              | 0               |
| 13       | 49     | -6                 | -7             | 0              | 0              | 0              | 0               |
| 14       | 15     | -226               | -226           | 0              | 0              | 0              | 0               |
| 14       | 46     | 125                | 125            | 0              | 0              | 0              | 0               |
| 15       | 45     | 65                 | 0              | 65             | 0              | 0              | 0               |
| 18       | 19     | -19                | 0              | 0              | 0              | -19            | 0               |
| 19       | 20     | -22                | 0              | 0              | 0              | -22            | 0               |
| 20       | 21     | -25                | 0              | 0              | 0              | -25            | 0               |
| 21       | 22     | -25                | 0              | 0              | 0              | -25            | 0               |
| 22       | 23     | 75                 | 70             | 0              | 0              | 5              | 0               |
| 22       | 38     | -100               | -70            | 0              | 0              | -30            | 0               |
| 23       | 24     | 66                 | 63             | 0              | 0              | 3              | 0               |
| 24       | 25     | 48                 | 31             | 0              | 0              | 16.5           | 0               |

| 24       | 25     | 48                    | 31                | 0                 | 0                 | 16.5              | 0                  |
|----------|--------|-----------------------|-------------------|-------------------|-------------------|-------------------|--------------------|
| 24       | 26     | -30                   | 0                 | 0                 | 0                 | -30               | 0                  |
| 25       | 30     | 90                    | 57                | 0                 | 0                 | 33                | 0                  |
| 26       | 27     | -30                   | 0                 | 0                 | 0                 | -30               | 0                  |
| 27       | 28     | -40                   | 0                 | 0                 | 0                 | -40               | 0                  |
| 28       | 29     | -45                   | 0                 | 0                 | 0                 | -45               | 0                  |
| 29       | 52     | 20                    | 0                 | 0                 | 0                 | 20                | 0                  |
| 30       | 31     | 86                    | 53                | 0                 | 0                 | 33                | 0                  |
| 31       | 32     | 80                    | 47                | 0                 | 0                 | 33                | 0                  |
| 32       | 33     | 3.8                   | 3.8               | 0                 | 0                 | 0                 | 0                  |
| 32       | 34     | 62                    | 29                | 0                 | 0                 | 33                | 0                  |
| 34       | 35     | 62                    | 29                | 0                 | 0                 | 33                | 0                  |
| 35       | 36     | 56                    | 23                | 0                 | 0                 | 33                | 0                  |
| 36       | 37     | -33                   | 0                 | -33               | 0                 | 0                 | 0                  |
| 37       | 38     | -39                   | 0                 | -39               | 0                 | 0                 | 0                  |
| 36       | 40     | 89                    | 23                | 33                | 0                 | 33                | 0                  |
| 37       | 39     | 6                     | 0                 | 6                 | 0                 | 0                 | 0                  |
| 38       | 44     | -53                   | 0                 | -53               | 0                 | 0                 | 0                  |
| 38       | 48     | -70                   | -70               | 0                 | 0                 | 0                 | 0                  |
| 38       | 49     | -30                   | 0                 | 0                 | 0                 | -30               | 0                  |
| 39       | 57     | 6                     | 0                 | 6                 | 0                 | 0                 | 0                  |
| 40       | 56     | 89                    | 23                | 33                | 0                 | 33                | 0                  |
| 41       | 42     | -57                   | -18.4             | -13               | 0                 | -26.4             | 0                  |
| 41       | 43     | 30                    | 10.8              | 7                 | 0                 | 16                | 0                  |
| 44       | 45     | -65                   | 0                 | -65               | 0                 | 0                 | 0                  |
| 46       | 47     | 125                   | 125               | 0                 | 0                 | 0                 | 0                  |
| 47       | 48     | 94                    | 95                | 0                 | 0                 | 0                 | 0                  |
| From bus | To bus | Line flow (MW)        |                   |                   |                   |                   |                    |
|          |        | Aggregate<br>Schedule | Schedule<br>tag 1 | Schedule<br>tag 2 | Schedule<br>tag 3 | Schedule<br>tag 8 | Schedule<br>tag 12 |
| 48       | 49     | 24                    | 25                | 0                 | 0                 | 0                 | 0                  |
| 49       | 50     | -30                   | 0                 | 0                 | 0                 | -30               | 0                  |
| 50       | 51     | -51                   | 0                 | 0                 | 0                 | -51               | 0                  |
| 52       | 53     | 15                    | 0                 | 0                 | 0                 | 15                | 0                  |
| 53       | 54     | -14                   | 0                 | 0                 | 0                 | -14               | 0                  |
| 54       | 55     | -18                   | 0                 | 0                 | 0                 | -18               | 0                  |
| 56       | 41     | 16.2                  | 4.6               | 5                 | 0                 | 6.6               | 0                  |
| 56       | 42     | 64.8                  | 18.4              | 20                | 0                 | 26.4              | 0                  |
| 57       | 56     | 0.4                   | 0                 | 0.4               | 0                 | 0                 | 0                  |

TABLE F.4

Sample power flow input file for the IEEE 57 bus test system

# AMERICAN ELECTRIC POWER 57 BUS SYSTEM

## PARAMETER DATA

-999

## BUS DATA

|             |        |        |         |        |        |        |       |        |      |
|-------------|--------|--------|---------|--------|--------|--------|-------|--------|------|
| 1 AMOS      | 1      | 0      | 2       | 0.0000 | 0.00   | 23.00  | 17.00 | 690.00 | 0.00 |
|             | 1.0400 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 2 BAKER     | 1      | 0      | 3       | 0.0000 | 0.00   | 3.00   | 88.00 | 200.00 | 0.00 |
|             | 1.0100 | 50.00  | -17.00  | 0.0000 | 0.0000 | 0      |       |        |      |
| 3 CRAWFORD  | 1      | 0      | 2       | 0.0000 | 0.00   | 101.00 | 21.00 | 100.00 | 0.00 |
|             | 0.9850 | 60.00  | -10.00  | 0.0000 | 0.0000 | 0      |       |        |      |
| 4 DOYLE     | 1      | 0      | 0       | 0.0000 | 0.00   | 0.00   | 0.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 5 DAWSON    | 1      | 0      | 0       | 0.0000 | 0.00   | 13.00  | 4.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 6 ELLS      | 1      | 0      | 2       | 0.0000 | 0.00   | 75.00  | 2.00  | 0.00   | 0.00 |
|             | 0.9800 | 25.00  | -8.00   | 0.0000 | 0.0000 | 0      |       |        |      |
| 7 FARLIE    | 1      | 0      | 0       | 0.0000 | 0.00   | 0.00   | 0.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 8 GRANGE    | 1      | 0      | 2       | 0.0000 | 0.00   | 150.00 | 22.00 | 450.00 | 0.00 |
|             | 1.0050 | 200.00 | -140.00 | 0.0000 | 0.0000 | 0      |       |        |      |
| 9 HOMER     | 1      | 0      | 2       | 0.0000 | 0.00   | 121.00 | 26.00 | 0.00   | 0.00 |
|             | 0.9800 | 9.00   | -3.00   | 0.0000 | 0.0000 | 0      |       |        |      |
| 10 JENKINS  | 1      | 0      | 0       | 0.0000 | 0.00   | 32.00  | 2.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 11 KINCAID  | 1      | 0      | 0       | 0.0000 | 0.00   | 100.00 | 0.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 12 LOESCHER | 1      | 0      | 2       | 0.0000 | 0.00   | 377.00 | 24.00 | 275.00 | 0.00 |
|             | 1.0150 | 155.00 | -50.00  | 0.0000 | 0.0000 | 0      |       |        |      |
| 13 MOSES    | 1      | 0      | 0       | 0.0000 | 0.00   | 18.00  | 2.30  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 14 NESTLE   | 1      | 0      | 0       | 0.0000 | 0.00   | 10.50  | 5.30  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 15 OAKDALE  | 1      | 0      | 0       | 0.0000 | 0.00   | 22.00  | 5.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 16 ANDY     | 1      | 0      | 0       | 0.0000 | 0.00   | 43.00  | 3.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 17 ANDY     | 1      | 0      | 0       | 0.0000 | 0.00   | 42.00  | 8.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 18 DOYLE    | 1      | 0      | 0       | 0.0000 | 0.00   | 27.20  | 9.80  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.1000 | 0      |       |        |      |
| 19 RICHTER  | 1      | 0      | 0       | 0.0000 | 0.00   | 3.30   | 0.60  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 20 RICHTER  | 1      | 0      | 0       | 0.0000 | 0.00   | 2.30   | 1.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 21 RICHTER  | 1      | 0      | 0       | 0.0000 | 0.00   | 0.00   | 0.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 22 BEAVER2  | 1      | 0      | 0       | 0.0000 | 0.00   | 0.00   | 0.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 23 BEAVER1  | 1      | 0      | 0       | 0.0000 | 0.00   | 6.30   | 2.10  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 24 POOL     | 1      | 0      | 0       | 0.0000 | 0.00   | 0.00   | 0.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 25 POOLA    | 1      | 0      | 0       | 0.0000 | 0.00   | 6.30   | 3.20  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0590 | 0      |       |        |      |
| 26 POOL     | 1      | 0      | 0       | 0.0000 | 0.00   | 0.00   | 0.00  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 27 HAMEL    | 1      | 0      | 0       | 0.0000 | 0.00   | 9.30   | 0.50  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 28 WYNCOTE  | 1      | 0      | 0       | 0.0000 | 0.00   | 4.60   | 2.30  | 0.00   | 0.00 |
|             | 0.0000 | 0.00   | 0.00    | 0.0000 | 0.0000 | 0      |       |        |      |
| 29 FARLIE   | 1      | 0      | 0       | 0.0000 | 0.00   | 52.00  | 2.60  | 0.00   | 0.00 |

|              |      |      |        |        |      |       |       |      |      |
|--------------|------|------|--------|--------|------|-------|-------|------|------|
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 30 CHESTER   | 1    | 0    | 0      | 0.0000 | 0.00 | 3.60  | 1.80  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 31 HANOVER   | 1    | 0    | 0      | 0.0000 | 0.00 | 5.80  | 2.90  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 32 UXBRIDGE  | 1    | 0    | 0      | 0.0000 | 0.00 | 14.00 | 0.80  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 33 LUXBRIDGE | 1    | 0    | 0      | 0.0000 | 0.00 | 3.80  | 1.90  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 34 UXBRIDGE  | 1    | 0    | 0      | 0.0000 | 0.00 | 0.00  | 0.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 35 COPLEYMN  | 1    | 0    | 0      | 0.0000 | 0.00 | 6.00  | 3.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 36 COPLEY    | 1    | 0    | 0      | 0.0000 | 0.00 | 0.00  | 0.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 37 NCOPLEY   | 1    | 0    | 0      | 0.0000 | 0.00 | 0.00  | 0.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 38 STANTON   | 1    | 0    | 0      | 0.0000 | 0.00 | 14.00 | 7.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 39 WTAUNTON  | 1    | 0    | 0      | 0.0000 | 0.00 | 0.00  | 0.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 40 TAUNTON   | 1    | 0    | 0      | 0.0000 | 0.00 | 0.00  | 0.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 41 KINCAID   | 1    | 0    | 0      | 0.0000 | 0.00 | 6.30  | 3.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 42 AIRPORT3  | 1    | 0    | 0      | 0.0000 | 0.00 | 7.10  | 4.40  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 43 KINCAID   | 1    | 0    | 0      | 0.0000 | 0.00 | 2.00  | 1.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 44 SOAKDALE  | 1    | 0    | 0      | 0.0000 | 0.00 | 12.00 | 1.80  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 45 OAKDALE   | 1    | 0    | 0      | 0.0000 | 0.00 | 0.00  | 0.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 46 NESTLE    | 1    | 0    | 0      | 0.0000 | 0.00 | 0.00  | 0.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 47 AIRPORT2  | 1    | 0    | 0      | 0.0000 | 0.00 | 29.70 | 11.60 | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 48 AIRPORT1  | 1    | 0    | 0      | 0.0000 | 0.00 | 0.00  | 0.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 49 MOSES     | 1    | 0    | 0      | 0.0000 | 0.00 | 18.00 | 8.50  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 50 MANX      | 1    | 0    | 0      | 0.0000 | 0.00 | 21.00 | 10.50 | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 51 JENKINS   | 1    | 0    | 0      | 0.0000 | 0.00 | 18.00 | 5.30  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 52 NVEXLEY   | 1    | 0    | 0      | 0.0000 | 0.00 | 4.90  | 2.20  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 53 VEXLEY    | 1    | 0    | 0      | 0.0000 | 0.00 | 30.00 | 10.00 | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0630 | 0    |       |       |      |      |
| 54 VEXLEYSQ  | 1    | 0    | 0      | 0.0000 | 0.00 | 4.10  | 1.40  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 55 HOMER     | 1    | 0    | 0      | 0.0000 | 0.00 | 6.80  | 3.40  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 56 WTAUNTON  | 1    | 0    | 0      | 0.0000 | 0.00 | 7.60  | 2.20  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |
| 57 WTAUNTON  | 1    | 0    | 0      | 0.0000 | 0.00 | 6.70  | 2.00  | 0.00 | 0.00 |
| 0.0000       | 0.00 | 0.00 | 0.0000 | 0.0000 | 0    |       |       |      |      |

-999

#### BRANCH DATA

|   |    |   |   |   |   |          |          |         |   |   |   |   |   |
|---|----|---|---|---|---|----------|----------|---------|---|---|---|---|---|
| 1 | 2  | 1 | 0 | 1 | 0 | 0.008300 | 0.028000 | 0.12900 | 0 | 0 | 0 | 0 | 0 |
| 1 | 15 | 1 | 1 | 1 | 0 | 0.016600 | 0.056000 | 0.25800 | 0 | 0 | 0 | 0 | 0 |
| 1 | 16 | 1 | 0 | 1 | 0 | 0.045400 | 0.206000 | 0.05460 | 0 | 0 | 0 | 0 | 0 |

|        |      |        |         |         |         |          |          |         |        |        |   |   |   |
|--------|------|--------|---------|---------|---------|----------|----------|---------|--------|--------|---|---|---|
| 1      | 17   | 1      | 0       | 1       | 0       | 0.023800 | 0.108000 | 0.02860 | 0      | 0      | 0 | 0 | 0 |
| 2      | 3    | 1      | 0       | 1       | 0       | 0.029800 | 0.085000 | 0.08180 | 0      | 0      | 0 | 0 | 0 |
| 3      | 4    | 1      | 0       | 1       | 0       | 0.011200 | 0.036600 | 0.03800 | 0      | 0      | 0 | 0 | 0 |
| 3      | 15   | 1      | 0       | 1       | 0       | 0.016200 | 0.053000 | 0.05440 | 0      | 0      | 0 | 0 | 0 |
| 4      | 5    | 1      | 0       | 1       | 0       | 0.062500 | 0.132000 | 0.02580 | 0      | 0      | 0 | 0 | 0 |
| 4      | 6    | 1      | 0       | 1       | 0       | 0.043000 | 0.148000 | 0.03480 | 0      | 0      | 0 | 0 | 0 |
| 4      | 18   | 1      | 0       | 1       | 1       | 0.000000 | 0.555000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 0.9700 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 4      | 18   | 1      | 0       | 2       | 1       | 0.000000 | 0.430000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 0.9780 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 5      | 6    | 1      | 0       | 1       | 0       | 0.030200 | 0.064100 | 0.01240 | 0      | 0      | 0 | 0 | 0 |
| 6      | 7    | 1      | 0       | 1       | 0       | 0.020000 | 0.102000 | 0.02760 | 0      | 0      | 0 | 0 | 0 |
| 6      | 8    | 1      | 0       | 1       | 0       | 0.033900 | 0.173000 | 0.04700 | 0      | 0      | 0 | 0 | 0 |
| 7      | 8    | 1      | 0       | 1       | 0       | 0.013900 | 0.071200 | 0.01940 | 0      | 0      | 0 | 0 | 0 |
| 7      | 29   | 1      | 0       | 1       | 1       | 0.000000 | 0.064800 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 0.9670 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 8      | 9    | 1      | 0       | 1       | 0       | 0.009900 | 0.050500 | 0.05480 | 0      | 0      | 0 | 0 | 0 |
| 9      | 10   | 1      | 0       | 1       | 0       | 0.0300   | 0.167900 | 0.04400 | 0      | 0      | 0 | 0 | 0 |
| 9      | 11   | 1      | 0       | 1       | 0       | 0.025800 | 0.084800 | 0.02180 | 0      | 0      | 0 | 0 | 0 |
| 9      | 12   | 1      | 0       | 1       | 0       | 0.064800 | 0.295000 | 0.07720 | 0      | 0      | 0 | 0 | 0 |
| 9      | 13   | 1      | 0       | 1       | 0       | 0.048100 | 0.158000 | 0.04060 | 0      | 0      | 0 | 0 | 0 |
| 9      | 55   | 1      | 0       | 1       | 1       | 0.000000 | 0.120500 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 0.9400 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 10     | 12   | 1      | 0       | 1       | 0       | 0.027700 | 0.126200 | 0.03280 | 0      | 0      | 0 | 0 | 0 |
| 10     | 51   | 1      | 0       | 1       | 1       | 0.000000 | 0.071200 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 0.9300 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 11     | 13   | 1      | 0       | 1       | 0       | 0.022300 | 0.073200 | 0.01880 | 0      | 0      | 0 | 0 | 0 |
| 11     | 41   | 1      | 0       | 1       | 1       | 0.000000 | 0.749000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 0.9550 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 11     | 43   | 1      | 0       | 1       | 1       | 0.000000 | 0.153000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 0.9580 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 12     | 13   | 1      | 0       | 1       | 0       | 0.017800 | 0.058000 | 0.06040 | 0      | 0      | 0 | 0 | 0 |
| 12     | 16   | 1      | 0       | 1       | 0       | 0.018000 | 0.081300 | 0.02160 | 0      | 0      | 0 | 0 | 0 |
| 12     | 17   | 1      | 0       | 1       | 0       | 0.039700 | 0.179000 | 0.04760 | 0      | 0      | 0 | 0 | 0 |
| 13     | 14   | 1      | 0       | 1       | 0       | 0.013200 | 0.043400 | 0.01100 | 0      | 0      | 0 | 0 | 0 |
| 13     | 15   | 1      | 0       | 1       | 0       | 0.0200   | 0.0800   | 0.02300 | 0      | 0      | 0 | 0 | 0 |
| 13     | 49   | 1      | 0       | 1       | 1       | 0.000000 | 0.191000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 0.8950 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 14     | 15   | 1      | 0       | 1       | 0       | 0.017100 | 0.054700 | 0.01480 | 0      | 0      | 0 | 0 | 0 |
| 14     | 46   | 1      | 0       | 1       | 1       | 0.000000 | 0.073500 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 0.9000 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 15     | 45   | 1      | 0       | 1       | 1       | 0.000000 | 0.104200 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 0.9550 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 18     | 19   | 1      | 0       | 1       | 0       | 0.461000 | 0.685000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 19     | 20   | 1      | 0       | 1       | 0       | 0.283000 | 0.434000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 20     | 21   | 1      | 0       | 1       | 1       | 0.000000 | 0.776700 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 1.0430 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 21     | 22   | 1      | 0       | 1       | 0       | 0.073600 | 0.117000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 22     | 23   | 1      | 0       | 1       | 0       | 0.009900 | 0.015200 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 22     | 38   | 1      | 0       | 1       | 0       | 0.019200 | 0.029500 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 23     | 24   | 1      | 0       | 1       | 0       | 0.166000 | 0.256000 | 0.00840 | 0      | 0      | 0 | 0 | 0 |
| 24     | 25   | 1      | 0       | 1       | 1       | 0.000000 | 1.182000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 1.0000 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 24     | 25   | 1      | 0       | 2       | 1       | 0.000000 | 1.230000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 1.0000 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 24     | 26   | 1      | 0       | 1       | 1       | 0.000000 | 0.047300 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 1.0430 | 0.00 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000  | 0.00000  | 0.00000 | 0.0000 | 0.0000 |   |   |   |
| 25     | 30   | 1      | 0       | 1       | 0       | 0.135000 | 0.202000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 26     | 27   | 1      | 0       | 1       | 0       | 0.165000 | 0.254000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 27     | 28   | 1      | 0       | 1       | 0       | 0.061800 | 0.095400 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 28     | 29   | 1      | 0       | 1       | 0       | 0.041800 | 0.058700 | 0.00000 | 0      | 0      | 0 | 0 | 0 |
| 29     | 52   | 1      | 0       | 1       | 0       | 0.144200 | 0.187000 | 0.00000 | 0      | 0      | 0 | 0 | 0 |

|        |      |        |          |          |          |          |          |          |        |        |   |   |   |
|--------|------|--------|----------|----------|----------|----------|----------|----------|--------|--------|---|---|---|
| 30     | 31   | 1      | 0        | 1        | 0        | 0.326000 | 0.497000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 31     | 32   | 1      | 0        | 1        | 0        | 0.507000 | 0.755000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 32     | 33   | 1      | 0        | 1        | 0        | 0.039200 | 0.036000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 32     | 34   | 1      | 0        | 1        | 1        | 0.000000 | 0.953000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 0.9750 | 0.00 | 0.0000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.0000 | 0.0000 |   |   |   |
| 34     | 35   | 1      | 0        | 1        | 0        | 0.052000 | 0.078000 | 0.00320  | 0      | 0      | 0 | 0 | 0 |
| 35     | 36   | 1      | 0        | 1        | 0        | 0.043000 | 0.053700 | 0.00160  | 0      | 0      | 0 | 0 | 0 |
| 36     | 37   | 1      | 0        | 1        | 0        | 0.029000 | 0.036600 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 37     | 38   | 1      | 0        | 1        | 0        | 0.065100 | 0.100900 | 0.00200  | 0      | 0      | 0 | 0 | 0 |
| 36     | 40   | 1      | 0        | 1        | 0        | 0.030000 | 0.046600 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 37     | 39   | 1      | 0        | 1        | 0        | 0.023900 | 0.037900 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 38     | 44   | 1      | 0        | 1        | 0        | 0.028900 | 0.058500 | 0.00200  | 0      | 0      | 0 | 0 | 0 |
| 38     | 48   | 1      | 0        | 1        | 0        | 0.031200 | 0.048200 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 38     | 49   | 1      | 0        | 1        | 0        | 0.115000 | 0.177000 | 0.00600  | 0      | 0      | 0 | 0 | 0 |
| 39     | 57   | 1      | 0        | 1        | 1        | 0.000000 | 1.355000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 0.9800 | 0.00 | 0.0000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.0000 | 0.0000 |   |   |   |
| 40     | 56   | 1      | 0        | 1        | 1        | 0.000000 | 1.195000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 0.9580 | 0.00 | 0.0000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.0000 | 0.0000 |   |   |   |
| 41     | 42   | 1      | 0        | 1        | 0        | 0.207000 | 0.352000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 41     | 43   | 1      | 0        | 1        | 0        | 0.000000 | 0.412000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 44     | 45   | 1      | 0        | 1        | 0        | 0.062400 | 0.124200 | 0.00400  | 0      | 0      | 0 | 0 | 0 |
| 46     | 47   | 1      | 0        | 1        | 0        | 0.023000 | 0.068000 | 0.00320  | 0      | 0      | 0 | 0 | 0 |
| 47     | 48   | 1      | 0        | 1        | 0        | 0.018200 | 0.023300 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 48     | 49   | 1      | 0        | 1        | 0        | 0.083400 | 0.129000 | 0.00480  | 0      | 0      | 0 | 0 | 0 |
| 49     | 50   | 1      | 0        | 1        | 0        | 0.080100 | 0.128000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 50     | 51   | 1      | 0        | 1        | 0        | 0.600    | 0.220000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 52     | 53   | 1      | 0        | 1        | 0        | 0.076200 | 0.098400 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 53     | 54   | 1      | 0        | 1        | 0        | 0.187800 | 0.232000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 54     | 55   | 1      | 0        | 1        | 0        | 0.173200 | 0.226500 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 56     | 41   | 1      | 0        | 1        | 0        | 0.553000 | 0.549000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 56     | 42   | 1      | 0        | 1        | 0        | 0.212500 | 0.354000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| 57     | 56   | 1      | 0        | 1        | 0        | 0.174000 | 0.260000 | 0.000000 | 0      | 0      | 0 | 0 | 0 |
| -999   |      |        |          |          |          |          |          |          |        |        |   |   |   |